Exercice 1

a) $2^{-1} = \frac{1}{2}$ b) $4^{-2} = \frac{1}{16}$ c) $\frac{1}{8}^{-2} = 64$ d) $\left(\frac{64}{27}\right)^{\frac{1}{3}} = \frac{4}{3}$ e) $125^{-\frac{1}{3}} = \frac{1}{5}$

f) $0{,}125^{-\frac{1}{3}} = \frac{1}{8}^{-\frac{1}{3}} = 8^{\frac{1}{3}} = 2$ g) $0{,}0001^{-\frac{3}{4}} = \frac{1}{10000}^{-\frac{3}{4}} = 10000^{\frac{3}{4}} = 10^3 = 1000$

h) $16^{-\frac{3}{4}} = 2^{-3} = \frac{1}{8}$ i) $\frac{1}{81}^{-\frac{3}{4}} = 81^{\frac{3}{4}} = 3^3 = 27$ j) $\left(3^{\frac{1}{2}} \cdot 3^{\frac{1}{4}}\right)^2 = \left(3^{\frac{3}{4}}\right)^2 = 3^{\frac{3}{2}} = \sqrt{27}$

Exercice 2

a) $\frac{1}{a} = a^{-1}$ b) $\sqrt[3]{a} = a^{\frac{1}{3}}$ c) $\sqrt{a^3} = a^{\frac{3}{2}}$ d) $\sqrt{\frac{a^2}{a^3}} = \sqrt{a^{-1}} = a^{-\frac{1}{2}}$

e) $\frac{(a^2)^3}{\sqrt{a}} = \frac{a^6}{a^{\frac{1}{2}}} = a^{\frac{11}{2}}$ f) $\sqrt{(a^3)^2} = a^3$ g) $\frac{1}{a^3 \cdot a^5} = a^{-8}$

h) $\left(a^2 \cdot a^{-3}\right)^{-\frac{1}{2}} = \left(a^{-1}\right)^{-\frac{1}{2}} = a^{\frac{1}{2}}$

i) $a\sqrt{a\sqrt{a\sqrt{a}}} = a\sqrt{a\sqrt{a \cdot a^{\frac{1}{2}}}} = a\sqrt{a\sqrt{a^{\frac{3}{2}}}} = a\sqrt{a \cdot a^{\frac{3}{4}}} = a\sqrt{a^{\frac{7}{4}}} = a \cdot a^{\frac{7}{8}} = a^{\frac{15}{8}}$

Exercice 3

a) $2^{-\frac{1}{3}} = \sqrt[3]{\frac{1}{2}}$ b)+c) $3^{-1} = \frac{1}{3}$ d) ok e) ok f) ok h) ok i) ok j) ok

k) $\left(a^2 \cdot b^2\right)^5 = a^{10} \cdot b^{10}$ m) ok n) ok o) $2^{\frac{1}{3}} \cdot 2^{\frac{5}{2}} = 2^{\frac{17}{6}}$ p) ok

q) ok r) $\sqrt{a^2 \cdot b^2} = a \cdot b$ s) ok t) ok

Exercice 4

a) $\dfrac{\sqrt{18}\cdot 2^3}{3^{\frac{1}{2}}} = \dfrac{\sqrt{2\cdot 9}\cdot 2^3}{3^{\frac{1}{2}}} = \dfrac{2^{\frac{1}{2}}\cdot 3\cdot 2^3}{3^{\frac{1}{2}}} = 3^{\frac{1}{2}}\cdot 2^{\frac{7}{2}} = 8\cdot\sqrt{6}$

b) $\dfrac{\sqrt{2}\,(1+\sqrt{2})}{2^{½}+2} = \dfrac{\sqrt{2}\,(1+\sqrt{2})}{\sqrt{2}\,(1+\sqrt{2})} = 1$

c) $\dfrac{2^{-\frac{1}{3}}+2^{\frac{5}{3}}}{2^{\frac{2}{3}}} = \dfrac{2^{\frac{2}{3}}\,(2^{-1}+2^{\frac{3}{3}})}{2^{\frac{2}{3}}} = 2^{-1}+2 = \dfrac{5}{2}$

d) $\dfrac{\sqrt{252}+7^{½}}{2^3-1} = \dfrac{\sqrt{7\cdot 36}+\sqrt{7}}{7} = \dfrac{\sqrt{7}\,(6+1)}{7} = \sqrt{7}$

Exercice 5

a) $\sqrt{2}-\sqrt{8}+\dfrac{3}{\sqrt{2}}-\dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\,(2-\sqrt{16}+3-1) = 0$

b) $\left(3-\sqrt{5}\right)^2-\left(3+\sqrt{5}\right)^2+\sqrt{720} = \left(9-6\sqrt{5}+5\right)-\left(9+6\sqrt{5}+5\right)+\sqrt{720} =$

$-12\sqrt{5}+\sqrt{720} = -\sqrt{144}\sqrt{5}+\sqrt{720} = 0$

c) $\left(5\sqrt{3}\right)^3-375\sqrt{3} = \left(5^3\cdot 3\sqrt{3}\right)-375\sqrt{3} = 0$

d) $\left(\sqrt{2}+\sqrt{3}\right)^2-\left(2+\sqrt{3}\right)\left(\sqrt{3}-2\right)+\left(1-\sqrt{6}\right)\left(\sqrt{2}+\sqrt{27}\right)\left(5+2\sqrt{6}\right)-(3-4)+\left(1-\sqrt{6}\right)\left(\sqrt{2}+\sqrt{27}\right)$

=

$\left(5+2\sqrt{6}\right)+1+\left(\sqrt{2}+\sqrt{27}-\sqrt{12}-\sqrt{162}\right) = 6+2\sqrt{6}+\left(\sqrt{2}+3\sqrt{3}-2\sqrt{3}-9\sqrt{2}\right)$

=

$6+2\sqrt{6}+\left(\sqrt{3}-8\sqrt{2}\right) = 6+2\sqrt{6}+\sqrt{3}-8\sqrt{2}$

Exercice 6

a) $0{,}001^{-\frac{2}{3}} = 100$

b) $\left(\dfrac{16}{25}\right)^{\frac{3}{2}} = \dfrac{64}{125}$

c) $0{,}125^{-\frac{4}{3}} = 16$

d) $0{,}25^{\frac{3}{2}} = \dfrac{1}{8}$

e) $81^{1{,}25} = 81^{\frac{5}{4}} = 3^5 = 243$

f) $32^{-\frac{7}{5}} = 2^{-7} = \dfrac{1}{128}$

Exercice 7

a) $\dfrac{a^2\cdot a^3}{\sqrt{a}\cdot\sqrt[3]{a}} = \dfrac{a^5}{a^{\frac{1}{2}}\cdot a^{\frac{1}{3}}} = \dfrac{a^5}{a^{\frac{5}{6}}} = a^{\frac{25}{6}}$

b) $\sqrt{a^3\cdot\sqrt[3]{a}} = \sqrt{a^{\frac{10}{3}}} = a^{\frac{5}{3}}$

c) $\sqrt{\dfrac{\sqrt{a}}{a^{-2}}} = \sqrt{a^{\frac{1}{2}}\cdot a^2} = a^{\frac{5}{4}}$

d) $a^{½}\cdot\sqrt{a}\cdot\sqrt{\sqrt{a}}\cdot a^{¼} = a^{½}\cdot a^{½}\cdot\sqrt{a^{½}}\cdot a^{¼} = a\cdot a^{½} = a^{\frac{3}{2}}$

e) $a\sqrt{a\sqrt{a\sqrt{\sqrt{a}\cdot a}}} = a\sqrt{a\sqrt{a\sqrt{a^{½}\cdot a}}} = a\sqrt{a\sqrt{a\sqrt{a^{\frac{3}{2}}}}} = a\sqrt{a\sqrt{a\cdot a^{\frac{3}{4}}}} = a\sqrt{a\sqrt{a^{\frac{7}{4}}}} = a\sqrt{a^{\frac{15}{8}}} = a^{\frac{31}{16}}$

Exercice 8

a) $\left(\sqrt{a^2\,b^3}\right)^6 = a^6\,b^9$ b) $\left(\frac{27x^3}{8a^{-3}}\right)^{\frac{2}{3}} = \frac{9x^2}{4a^{-2}} = \frac{9}{4}a^2x^2$ c) $\left(\frac{4a^{-2}}{9x^2}\right)^{-\frac{1}{2}} = \frac{2^{-1}a}{3^{-1}x^{-1}} = \frac{3ax}{2}$

d) $\sqrt{a^{-2}\cdot b}\cdot\sqrt[3]{a\cdot b^{-3}} = a^{-1}\,b^{\frac{1}{2}}\,a^{\frac{1}{3}}\,b^{-1} = a^{-\frac{2}{3}}\,b^{-\frac{1}{2}}$

e) $\frac{\sqrt{x\sqrt{y}}}{\sqrt{y\sqrt{x}}} = \frac{x^{\frac{1}{2}}\,y^{\frac{1}{4}}}{y^{\frac{1}{2}}\,x^{\frac{1}{4}}} = \frac{x^{\frac{1}{4}}}{y^{\frac{1}{4}}} = \sqrt[4]{\frac{x}{y}}$ f) $\left(x^{-\frac{1}{2}}\cdot\sqrt{\frac{1}{x}}\right)^{-1} = x^{\frac{1}{2}}\cdot\sqrt{x} = x$

Exercice 9

a) $2^x = 4\sqrt{2} = 2^2\,2^{½} = 2^{\frac{5}{2}} \Rightarrow x = \frac{5}{2}$ b) $2^x = \sqrt[3]{\frac{\sqrt{2}}{64}} = \sqrt[3]{\frac{2^{\frac{1}{2}}}{2^6}} = \sqrt[3]{2^{-\frac{11}{2}}} = 2^{-\frac{11}{6}} \Rightarrow x = -\frac{11}{6}$

c) $5^{2x+1} = \sqrt{5\sqrt{125}} = \left(5\sqrt{5^3}\right)^{\frac{1}{2}} = \left(5\cdot 5^{\frac{3}{2}}\right)^{\frac{1}{2}} = \left(5^{\frac{5}{2}}\right)^{\frac{1}{2}} = 5^{\frac{5}{4}} \Rightarrow x = \frac{1}{8}$

d) $3^x\sqrt{3} = 9 \Rightarrow 3^{x+\frac{1}{2}} = 3^2 \Rightarrow x = \frac{3}{2}$

e) $2^x\cdot\sqrt[x]{64} = 32 \Rightarrow 2^x\cdot 2^{\frac{6}{x}} = 2^5 \Rightarrow x + \frac{6}{x} = 5$ $\Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 3\,;\,2$

Exercice 10

a) $\frac{\sqrt{2}\cdot 4}{\sqrt{8}} = \frac{2^{\frac{1}{2}}\cdot 2^2}{2^{\frac{3}{2}}} = 2$ b) $\frac{8^{-\frac{4}{3}}\cdot 4^{\frac{3}{2}}}{4\cdot\sqrt{2^6}} = \frac{2^{-4}\cdot 2^3}{2^2\cdot 2^3} = 2^{-6}$ c) $\frac{\sqrt{3}\cdot\frac{1}{\sqrt{9}}}{\sqrt{27}}\cdot 81 = \frac{3^{\frac{1}{2}}\cdot 3^{-1}\cdot 3^4}{3^{\frac{3}{2}}} = 3^2$

Exercice 12

a) $x = -1$ b) $x = -\frac{1}{3}$ c) $x = \frac{7}{2}$ d) $x = \frac{1}{5}$ e) $x = \frac{5}{4}$ f) $x = \frac{3}{2}$ g) $x = 1 - \frac{2}{5} = \frac{3}{5}$

h) $\sqrt{a^{\frac{2}{3}}:\sqrt[3]{a^{\frac{1}{2}}}\cdot\left(\left(a^{\frac{2}{3}}\right)^2\right)^{\frac{1}{3}}} = \left(a^{\frac{2}{3}}:a^{\frac{1}{6}}\right)^{\frac{1}{2}}\cdot\left(a^{\frac{4}{3}}\right)^{\frac{1}{3}} = \left(a^{\frac{2}{3}}\cdot a^{-\frac{1}{6}}\right)^{\frac{1}{2}}\cdot a^{\frac{4}{9}} = \left(a^{\frac{3}{6}}\right)^{\frac{1}{2}}\cdot a^{\frac{4}{9}} = a^{\frac{1}{4}}\cdot a^{\frac{4}{9}} = a^{\frac{25}{36}} \Rightarrow x = \frac{25}{36}$

Exercice 13

a) 9 b) $\frac{125}{8}$ c) $\frac{1}{25}^{-\frac{1}{2}} = 5$ d) $8^{\frac{1}{5}} : 4^{-\frac{1}{5}} = 2^{\frac{3}{5}} : 2^{-\frac{2}{5}} = 2^{\frac{3}{5}}\cdot 2^{\frac{2}{5}} = 2$ e) $16^{-0,75} = 16^{-\frac{3}{4}} = 2^{-3} = \frac{1}{8}$

Exercice 11

a) $\frac{\sqrt{7}}{\sqrt{21}} = \frac{\sqrt{7}\,\sqrt{21}}{\sqrt{21}\,\sqrt{21}} = \frac{\sqrt{7}\,\sqrt{7}\,\sqrt{3}}{21} = \frac{7\,\sqrt{3}}{7 \cdot 3}$ b) $\frac{\sqrt{32}}{1 - \sqrt{2}} = \frac{\sqrt{2}}{1 - \sqrt{2}} \cdot \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{\sqrt{2} + 2}{1 - 2} = -(2 + \sqrt{2})$

c) $\frac{\sqrt{6} + \sqrt{3}}{\sqrt{6} - \sqrt{3}} = \frac{\sqrt{6} + \sqrt{3}}{\sqrt{6} - \sqrt{3}} \cdot \frac{\sqrt{6} + \sqrt{3}}{\sqrt{6} + \sqrt{3}} = \frac{6 + 2\,\sqrt{6}\,\sqrt{3} + 3}{6 - 3} = \frac{9 + 2\,\sqrt{18}}{3} = \frac{9 + 6\,\sqrt{2}}{3} = 3 + 2\,\sqrt{2}$

d) $\frac{\sqrt{2} + 1}{\sqrt{2} - 1} = \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \cdot \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = 3 + 2\,\sqrt{2}$ e) $\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} \cdot \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \frac{8 - 2\,\sqrt{15}}{2}$

f $\frac{1}{\sqrt{2} + \sqrt{3} - \sqrt{5}} = \frac{1}{\sqrt{2} + \sqrt{3} - \sqrt{5}} \cdot \frac{(\sqrt{2} + \sqrt{3}) + \sqrt{5}}{(\sqrt{2} + \sqrt{3}) + \sqrt{5}} = \frac{\sqrt{2} + \sqrt{3} + \sqrt{5}}{(5 + 2\sqrt{6}) - 5} = \frac{(\sqrt{2} + \sqrt{3} + \sqrt{5})\sqrt{6}}{2\,\sqrt{6}\,\sqrt{6}} = \frac{\sqrt{12} + \sqrt{18} + \sqrt{30}}{12}$