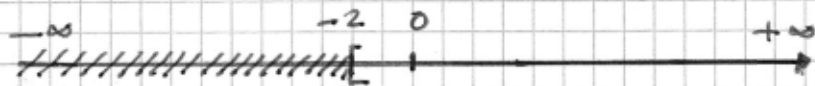


Chapitre 8. Inéquations - Corrigé

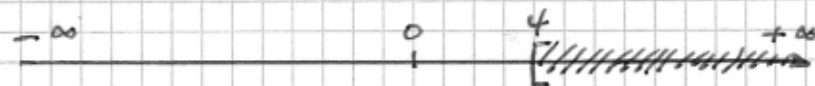
Exercice 1

1. $x < -2$



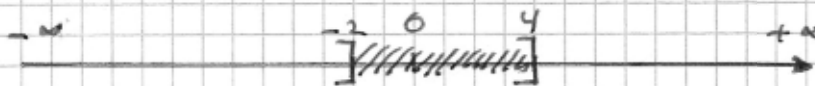
$$\Rightarrow x \in]-\infty; -2[.$$

2. $x \geq 4$



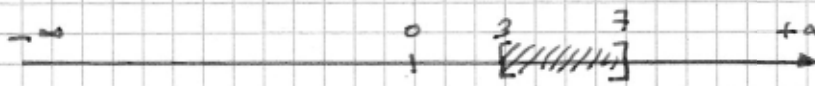
$$\Rightarrow x \in [4; +\infty[.$$

3. $-2 < x \leq 4$



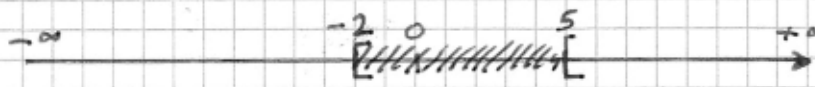
$$\Rightarrow x \in]-2; 4].$$

4. $3 \leq x \leq 7$



$$\Rightarrow x \in [3; 7].$$

5. $5 > x \geq -2$



$$\Rightarrow x \in [-2; 5].$$

Exercice 2

1. $] -5; 8]$: $-5 < x < 8$

2. $] -4; -1 [$: $-4 < x < -1$

3. $[4; \infty [$: $x \geq 4$

4. $] -\infty; -5 [$: $x < -5$

Exercice 3

$$1. \frac{4}{5}x - \frac{3}{4} > \frac{2}{3}x + \frac{1}{2}$$

$$\frac{48x}{60} - \frac{45}{60} > \frac{40x}{60} + \frac{30}{60}$$

$$48x - 45 > 40x + 30$$

$$8x > 75$$

$$x > \frac{75}{8}$$

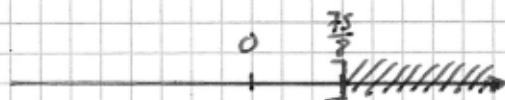
$$\Rightarrow x \in]\frac{75}{8}; +\infty[$$

D.C.

$$\cdot 60$$

$$-40x + 45$$

$$: 8$$



$$2. \frac{7}{2}x - \frac{x-1}{2} \leq x + \frac{3}{4}$$

$$\frac{14x}{4} - \left(\frac{2x-2}{4}\right) \leq \frac{4x}{4} + \frac{3}{4}$$

$$14x - (2x-2) \leq 4x + 3$$

$$14x - 2x + 2 \leq 4x + 3$$

$$12x + 2 \leq 4x + 3$$

$$8x \leq 1$$

$$x \leq \frac{1}{8}$$

$$\Rightarrow x \in]-\infty; \frac{1}{8}]$$

D.C.

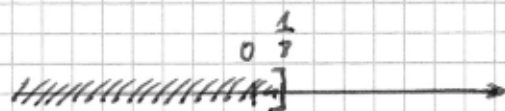
$$\cdot 4$$

$$D$$

$$R$$

$$-4x - 2$$

$$: 8$$



$$3. \frac{3x-2}{7} + \frac{x}{3} < x + \frac{4x+1}{21}$$

$$\frac{9x-6}{21} + \frac{7x}{21} < \frac{21x}{21} + \frac{4x+1}{21}$$

$$9x-6 + 7x < 21x + 4x+1$$

$$16x-6 < 25x+1$$

$$-9x < 7$$

$$x > -\frac{7}{9}$$

$$\Rightarrow x \in]-\frac{7}{9}; +\infty[$$

D.C.

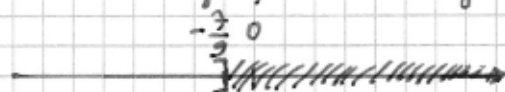
$$\cdot 21$$

$$R$$

$$-25x + 6$$

$$: (-9)$$

⚠ En multipliant ou divisant par un nombre négatif, on doit changer le sens de l'inégalité



$$4. 2(x-3) + 3x \geq 5x - \frac{1}{2}$$

$$2x - 6 + 3x \geq 5x - \frac{1}{2}$$

$$5x - 6 \geq 5x - \frac{1}{2}$$

$$-6 \geq -\frac{1}{2} \text{ impossible}$$

$$\Rightarrow x \in \emptyset \quad (\text{ensemble vide ; aucune solution})$$

D.C.

$$R$$

$$-5x$$

$$5. \quad x+3 - \frac{x+5}{4} < \frac{x-3}{5} - 2x$$

$$\frac{20x}{20} + \frac{60}{20} - \left(\frac{5x+25}{20}\right) < \frac{4x-12}{20} - \frac{40x}{20}$$

$$20x+60 - (5x+25) < 4x-12-40x$$

$$20x+60-5x-25 < 4x-12-40x$$

$$15x+35 < -36x-12$$

$$51x < -47$$

$$x < -\frac{47}{51}$$

$$p.c.$$

$$\cdot 20$$

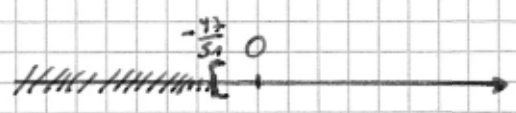
$$D$$

$$R$$

$$+26x-35$$

$$: 51$$

$$\Rightarrow x \in]-\infty; -\frac{47}{51}[$$



$$6. \quad x+19 - \frac{x}{3} \leq \frac{2x+57}{3}$$

$$3x+57-x \leq 2x+57$$

$$2x+57 \leq 2x+57$$

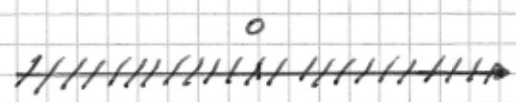
$$57 \leq 57 \quad \text{tajaras u par}$$

$$\cdot 3$$

$$R$$

$$-2x$$

$$\Rightarrow x \in \mathbb{R}$$



$$7. \quad 2x-1 < -4x$$

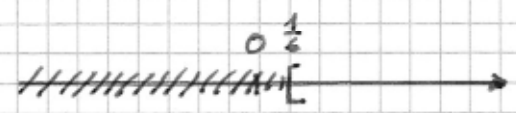
$$6x < 1$$

$$x < \frac{1}{6}$$

$$\Rightarrow x \in]-\infty; \frac{1}{6}[$$

$$+4x+1$$

$$: 6$$



$$8. \quad \frac{x}{3} + 2 < -3x + 12$$

$$x+6 < -9x+36$$

$$10x < 30$$

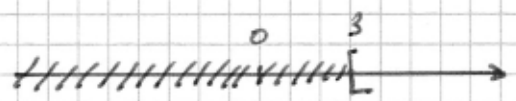
$$x < 3$$

$$\cdot 3$$

$$+9x-6$$

$$: 10$$

$$\Rightarrow x \in]-\infty; 3[$$



$$9. \quad \frac{x}{2} + 1 > -\frac{x-3}{2}$$

$$\frac{3x}{6} + \frac{6}{6} > -\left(\frac{2x-6}{6}\right)$$

$$3x+6 > -(2x-6)$$

$$3x+6 > -2x+6$$

$$5x > 0$$

$$x > 0$$

$$p.c.$$

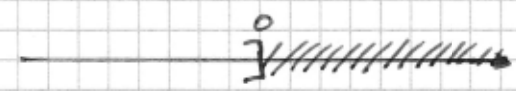
$$\cdot 6$$

$$D$$

$$+2x-6$$

$$: 5$$

$$\Rightarrow x \in]0; +\infty[$$



$$10. \frac{8-x}{2} < \frac{x+2}{3}$$

$$\frac{24-3x}{6} < \frac{2x+4}{6}$$

$$24-3x < 2x+4$$

$$-5x < -20$$

$$x > 4$$

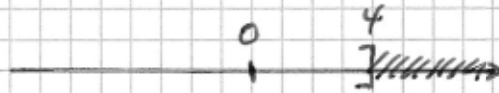
$$\Rightarrow x \in]4; +\infty[$$

D.C.

· 6

$$-2x - 24$$

$$: (-5) \quad \triangle$$



Exercice 4

$$\begin{array}{l|l}
 1. & -3 < 2x - 5 < 7 & +5 \\
 & 2 < 2x < 12 & :2 \\
 & \underline{1 < x < 6} & \\
 & \text{ou } x \in]1; 6[. &
 \end{array}$$

$$\begin{array}{l|l}
 2. & 3 \leq \frac{2-3x}{5} < 7 & \cdot 5 \\
 & 15 \leq 2-3x < 35 & -2 \\
 & 13 \leq -3x < 33 & :(-3) \quad \Delta \\
 & \underline{-\frac{13}{3} \geq x > -3} & \\
 & \text{ou } \underline{-3 < x \leq -\frac{13}{3}} & \text{ou } x \in]-\frac{13}{3}; -3].
 \end{array}$$

$$\begin{array}{l|l}
 3. & 0 \leq 4 - \frac{1}{3}x < 2 & \cdot 3 \\
 & 0 \leq 12 - x < 6 & -12 \\
 & -12 \leq -x < -6 & :(-1) \quad \Delta \\
 & \underline{12 \geq x > 6} & \\
 & \text{ou } \underline{6 < x \leq 12} & \text{ou } x \in]6; 12].
 \end{array}$$

$$\begin{array}{l|l}
 4. & 1 < \frac{2}{3}x + 1 < 2 & \cdot 3 \\
 & 3 < 2x + 3 < 6 & -3 \\
 & 0 < 2x < 3 & :2 \\
 & \underline{0 < x < \frac{3}{2}} & \\
 & \text{ou } x \in]0; \frac{3}{2}[. &
 \end{array}$$

$$\begin{array}{l|l}
 5. & 0 < 2x + 4 < 12 & -4 \\
 & -4 < 2x < 8 & :2 \\
 & \underline{-2 < x < 4} & \\
 & \text{ou } x \in]-2; 4[. &
 \end{array}$$

Exercice 5

$$\begin{array}{l|l|l}
 1. \quad 5x-10 < 4x & -4x+10 & 5+2x > 7 \\
 x < 10 & & 2x > 2 \\
 & & x > 1 \\
 \hline
 & & -5 \\
 & & : 2 \\
 \hline
 & &
 \end{array}$$

$$x < 10 \text{ et } x > 1 \Rightarrow \underline{1 < x < 10 \text{ ou } x \in [1; 10[}$$

$$\begin{array}{l|l|l}
 2. \quad 1-x > x-5 & -x-1 & x(2-x) < 4-x^2 \\
 -2x > -6 & : (-2) \Delta & 2x-x^2 < 4-x^2 \\
 x < 3 & & 2x < 4 \\
 & & x < 2 \\
 \hline
 & & \emptyset \\
 & & +x^2 \\
 & & : 2 \\
 \hline
 & &
 \end{array}$$

$$x < 3 \text{ et } x < 2 \Rightarrow \underline{x < 2 \text{ ou } x \in]-\infty; 2[}$$

$$\begin{array}{l|l|l}
 3. \quad (x-1)(x-2) > x^2-7 & \emptyset & (x-2)(x-3) < x^2+1 & \emptyset \\
 x^2-2x-x+2 > x^2-7 & R & x^2-3x-2x+6 < x^2+1 & R \\
 x^2-3x+2 > x^2-7 & -x^2 & x^2-5x+6 < x^2+1 & -x^2 \\
 -3x+2 > -7 & -2 & -5x+6 < 1 & -6 \\
 -3x > -9 & : (-3) \Delta & -5x < -5 & : (-5) \Delta \\
 x < 3 & & x > 1 &
 \end{array}$$

$$x < 3 \text{ et } x > 1 \Rightarrow \underline{1 < x < 3 \text{ ou } x \in]1; 3[}$$

$$\begin{array}{l|l|l}
 4. \quad 2x-1 > x+3 & -x+1 & \frac{4x}{3} + 3 < \frac{x+7}{2} & D.C. \\
 x > 4 & & \frac{8x}{6} + \frac{18}{6} < \frac{3x+14}{6} & .6 \\
 & & 8x+18 < 3x+14 & -3x-18 \\
 & & 5x < -4 & : 5 \\
 & & x < -\frac{4}{5} &
 \end{array}$$

$$x > 4 \text{ et } x < -\frac{4}{5} (= 0,6) \Rightarrow \underline{\text{impossible ou } x \in \emptyset}$$

$$\begin{array}{l|l|l}
 5. \quad 5(x-1) > 3x+5 & \emptyset & 3(x-1) < x-3 & \emptyset \\
 5x-5 > 3x+5 & -3x+5 & 3x-3 < x-3 & -x+3 \\
 2x > 10 & : 2 & 2x < 0 & : 2 \\
 x > 5 & & x < 0 &
 \end{array}$$

$$x > 5 \text{ et } x < 0 \Rightarrow \underline{\text{impossible ou } x \in \emptyset}$$

| | | | |
|--|---------------------------------------|---|-------------|
| $6. \quad \frac{2x+1}{5} \geq x - \frac{4x-5}{3}$ $\frac{6x+3}{15} \geq \frac{15x}{15} - \left(\frac{20x-25}{15}\right)$ $6x+3 \geq 15x - (20x-25)$ $6x+3 \geq 15x - 20x + 25$ $6x+3 \geq -5x + 25$ $11x \geq 22$ $x \geq 2$ | D.C. .15 D R +5x-3 :11 | $\frac{7x+2}{2} \geq 3x$ $7x+2 \geq 6x$ $x \geq -2$ | .2 -6x-2 |
| $x \geq 2 \text{ et } x \geq -2 \Rightarrow \underline{x \geq 2} \text{ ou } \underline{x \in [2; +\infty[}$ | | | |

| | | | |
|---|-------------------------------|---|--|
| $7. \quad x^2 - 3 > (x+2)(x-7)$ $x^2 - 3 > x^2 - 7x + 2x - 14$ $x^2 - 3 > x^2 - 5x - 14$ $-3 > -5x - 14$ $5x > -11$ $x > -\frac{11}{5}$ | D R -x^2 +5x+2 :5 | $\frac{x+48}{12} - \frac{x+8}{2} < \frac{3x}{7}$ $\frac{7x+336}{84} - \left(\frac{42x+336}{84}\right) < \frac{36x}{84}$ $7x+336 - (42x+336) < 36x$ $7x+336 - 42x - 336 < 36x$ $-35x < 36x$ $-71x < 0$ $x > 0$ | D.C. .84 D R -36x :(-71) Δ |
| $x > -\frac{11}{5} \text{ et } x > 0 \Rightarrow \underline{x > 0} \text{ ou } \underline{x \in]0; +\infty[}$ | | | |

| | | | |
|--|-----------|-------------------|----|
| $8. \quad 2x-3 > -x$ $3x > 3$ $x > 1$ | +x+3 : | $x-8 < 1$ $x < 9$ | +8 |
| $x > 1 \text{ et } x < 9 \Rightarrow \underline{1 < x < 9} \text{ ou } \underline{x \in]1; 9[}$ | | | |

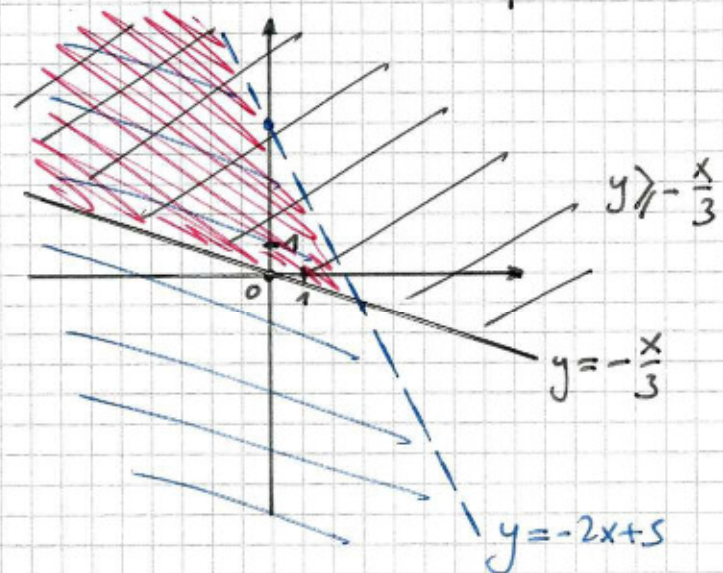
| | | | |
|--|------------------------|-------------------------------------|-----------|
| $9. \quad -2x+6 > x-9$ $-3x > -15$ $x < 5$ | -x-6 :(-3) Δ | $2(x+3) > x+6$ $2x+6 > x+6$ $x > 0$ | D -x-6 |
| $x < 5 \text{ et } x > 0 \Rightarrow \underline{0 < x < 5} \text{ ou } \underline{x \in]0; 5[}$ | | | |

Exercice 6

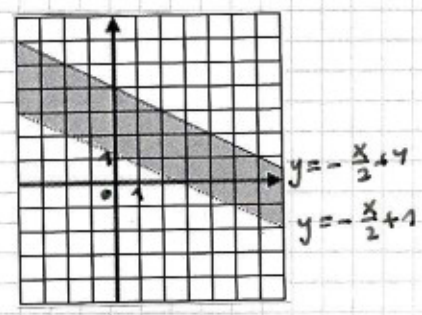
Dans chaque cas, la séquence est la suivante et ce pour chaque inéquation:

- 1) on isole y ;
- 2) on remplace l'inégalité par une égalité et on dessine la droite;
- 3) on revient à l'inégalité: si $y < \dots$, on est au-dessous de la droite et pas sur la droite (on dessine la droite en traitillés); si $y \leq \dots$, on est au-dessous ou sur la droite (on dessine la droite en trait plein); si $y > \dots$, on est au-dessus de la droite et pas sur la droite (on dessine la droite en traitillés); si $y \geq \dots$, on est au-dessus ou sur la droite (on dessine la droite en trait plein);
- 4) Le domaine défini par le système d'inéquations est l'intersection des zones déterminées en 3).

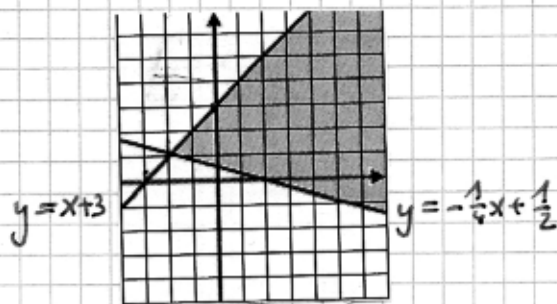
$$\begin{array}{l}
 1. \quad x + 3y \geq 0 \\
 \quad 3y \geq -x \\
 \quad y \geq -\frac{x}{3}
 \end{array}
 \left| \begin{array}{l} -x \\ :3 \end{array} \right.
 \begin{array}{l}
 2x + y - 5 < 0 \\
 y < -2x + 5
 \end{array}
 \left| \begin{array}{l} -2x + 5 \end{array} \right.$$



$$\begin{array}{l}
 2. \quad x \leq 8 - 2y \\
 \quad 2y \leq -x + 8 \\
 \quad y \leq -\frac{x}{2} + 4
 \end{array}
 \left| \begin{array}{l} +2y - x \\ :2 \end{array} \right.
 \begin{array}{l}
 x > 2 - 2y \\
 2y > -x + 2 \\
 y > -\frac{x}{2} + 1
 \end{array}
 \left| \begin{array}{l} +2y - x \\ :2 \end{array} \right.$$



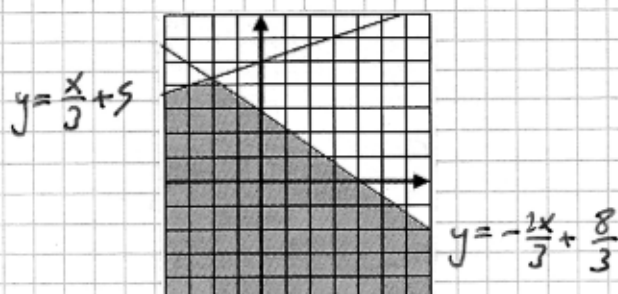
3.



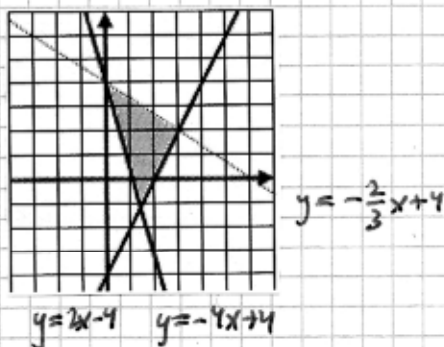
4. $x - 3y + 15 > 0$
 $-3y > -x - 15$
 $y < \frac{x}{3} + 5$

$-x - 15$
 $:(-3) \triangle$

$2x + 3y - 8 < 0$
 $3y < -2x + 8$
 $y < -\frac{2x}{3} + \frac{8}{3}$



5.



6. $x - 2y + 6 > 0$
 $-2y > -x - 6$
 $y < \frac{x}{2} + 3$

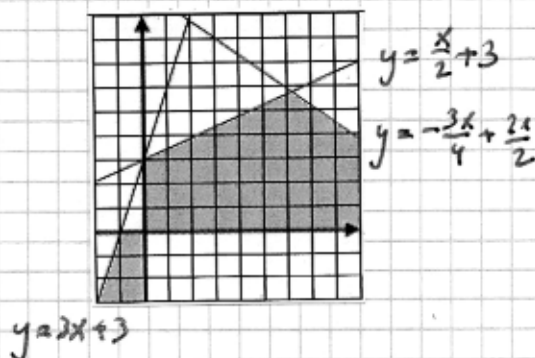
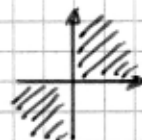
$-x - 6$
 $:(-2) \triangle$

$3x + 4y - 42 < 0$
 $4y < -3x + 42$
 $y < -\frac{3x}{4} + \frac{21}{2}$

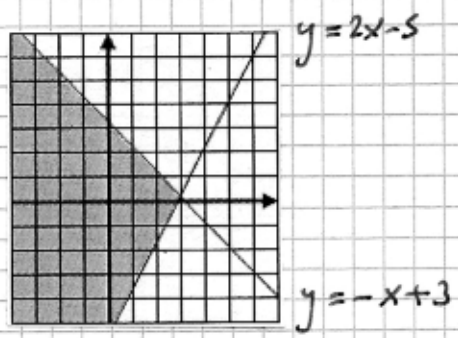
$3x - y + 3 > 0$
 $-y > -3x - 3$
 $y < 3x + 3$

$-3x - 3$
 $\cdot (-1) \triangle$

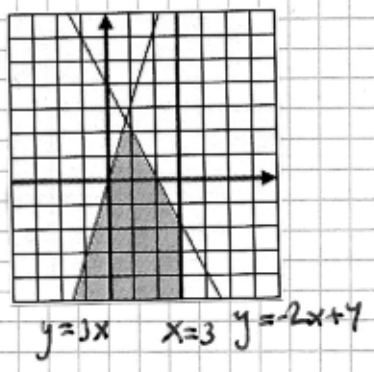
$x - y > 0$: on a 2 possibilités
 1) soit $x > 0$ et $y > 0$
 2) soit $x < 0$ et $y < 0$



7.

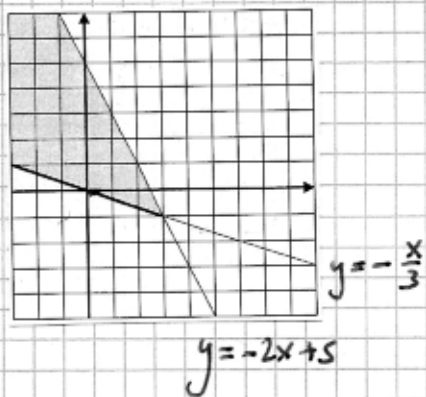


8.



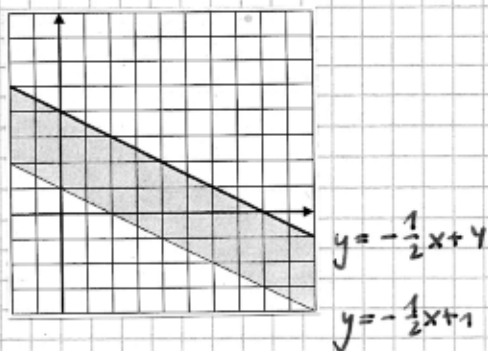
Ejercicio 7

1.



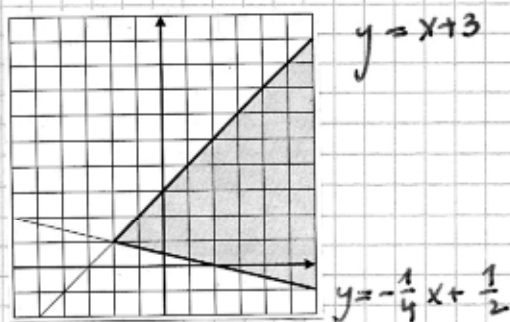
$$\Rightarrow \begin{cases} y < -2x + 5 \\ y \geq -\frac{x}{3} \end{cases}$$

2.



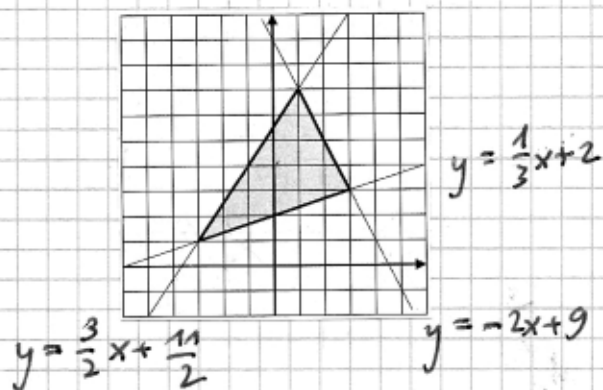
$$\Rightarrow \begin{cases} y \leq -\frac{1}{2}x + 4 \\ y > -\frac{1}{2}x + 1 \end{cases}$$

3.



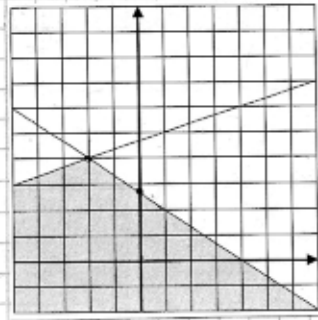
$$\Rightarrow \begin{cases} y \leq x + 3 \\ y \geq -\frac{1}{4}x + \frac{1}{2} \end{cases}$$

4.



$$\Rightarrow \begin{cases} y \geq \frac{1}{3}x + 2 \\ y \leq -2x + 9 \\ y \leq \frac{3}{2}x + \frac{11}{2} \end{cases}$$

5.



$$y = \frac{1}{3}x + \frac{14}{3}$$

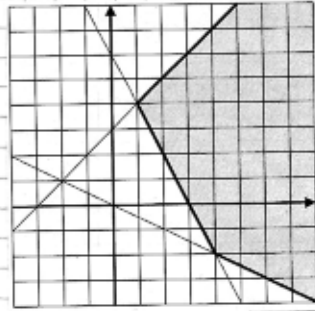
$$4 + \frac{2}{3} = \frac{14}{3}$$

$$y = -\frac{2}{3}x + \frac{8}{3}$$

$$2 + \frac{2}{3}$$

$$\rightarrow \begin{cases} y < \frac{1}{3}x + \frac{14}{3} \\ y < -\frac{2}{3}x + \frac{8}{3} \end{cases}$$

6.



$$y = x + 3$$

$$y = -2x + 6$$

$$y = -\frac{1}{2}x$$

$$\rightarrow \begin{cases} y \leq x + 3 \\ y \geq -2x + 6 \\ y \geq -\frac{1}{2}x \end{cases}$$
