

Chapitre 1. Equations du deuxieme degre - Corrigé

Exercice 1

1. $x^2 - 5x + 4 = 0$: $a=1, b=-5, c=4, b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 4 = 25 - 16 = 9 > 0$
 $\Rightarrow x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{5 + \sqrt{9}}{2} = \frac{5+3}{2} = \underline{4}$ et
 $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{5 - \sqrt{9}}{2} = \frac{5-3}{2} = \underline{1}.$
2. $x^2 = 4x \Rightarrow x^2 - 4x = 0$: $a=1, b=-4, c=0, b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot 0 = 16 > 0$
 $\Rightarrow x_1 = \frac{4 + \sqrt{16}}{2} = \frac{4+4}{2} = \underline{4}$ et
 $x_2 = \frac{4 - \sqrt{16}}{2} = \frac{4-4}{2} = \underline{0}.$
3. $x^2 + 7x + 12 = 0$: $a=1, b=7, c=12, b^2 - 4ac = 7^2 - 4 \cdot 1 \cdot 12 = 49 - 48 = 1 > 0$
 $\Rightarrow x_1 = \frac{-7 + \sqrt{1}}{2} = \frac{-7+1}{2} = \underline{-3}$ et
 $x_2 = \frac{-7 - \sqrt{1}}{2} = \frac{-7-1}{2} = \underline{-4}.$
4. $x^2 + x = 6 \Rightarrow x^2 + x - 6 = 0$: $a=1, b=1, c=-6, b^2 - 4ac = 1^2 - 4 \cdot 1 \cdot (-6) = 1 + 24 = 25 > 0$
 $\Rightarrow x_1 = \frac{-1 + \sqrt{25}}{2} = \frac{-1+5}{2} = \underline{2}$ et
 $x_2 = \frac{-1 - \sqrt{25}}{2} = \frac{-1-5}{2} = \underline{-3}.$
5. $x^2 + 9x + 18 = 0$: $a=1, b=9, c=18, b^2 - 4ac = 9^2 - 4 \cdot 1 \cdot 18 = 81 - 72 = 9 > 0$
 $\Rightarrow x_1 = \frac{-9 + \sqrt{9}}{2} = \frac{-9+3}{2} = \underline{-3}$ et
 $x_2 = \frac{-9 - \sqrt{9}}{2} = \frac{-9-3}{2} = \underline{-6}.$
6. $x^2 + 25 = 10x \Rightarrow x^2 - 10x + 25 = 0$: $a=1, b=-10, c=25, b^2 - 4ac = (-10)^2 - 4 \cdot 1 \cdot 25 = 100 - 100 = 0$
 $= 0 \Rightarrow x = \frac{10}{2 \cdot 1} = \underline{5}$ solution unique ($x = \frac{b}{2a}$)
7. $-x^2 + 6x - 5 = 0$: $a=-1, b=6, c=-5, b^2 - 4ac = 6^2 - 4 \cdot (-1) \cdot (-5) = 36 - 20 = 16 > 0$
 $\Rightarrow x_1 = \frac{-6 + \sqrt{16}}{2 \cdot (-1)} = \frac{-6+4}{-2} = \underline{1}$ et $x_2 = \frac{-6 - \sqrt{16}}{2 \cdot (-1)} = \frac{-6-4}{-2} = \underline{5}.$
8. $x^2 + 13x + 22 = 0$: $a=1, b=13, c=22, b^2 - 4ac = 13^2 - 4 \cdot 1 \cdot 22 = 169 - 88 = 81 > 0$
 $\Rightarrow x_1 = \frac{-13 + \sqrt{81}}{2} = \frac{-13+9}{2} = \underline{-2}$ et $x_2 = \frac{-13 - \sqrt{81}}{2} = \frac{-13-9}{2} = \underline{-11}.$
9. $x^2 - 7x - 18 = 0$: $a=1, b=-7, c=-18, b^2 - 4ac = (-7)^2 - 4 \cdot 1 \cdot (-18) = 49 + 72 = 121 > 0$
 $\Rightarrow x_1 = \frac{7 + \sqrt{121}}{2} = \frac{7+11}{2} = \underline{9}$ et $x_2 = \frac{7 - \sqrt{121}}{2} = \frac{7-11}{2} = \underline{-2}.$
10. $2x^2 + 7x + 3 = 0$: $a=2, b=7, c=3, b^2 - 4ac = 7^2 - 4 \cdot 2 \cdot 3 = 49 - 24 = 25 > 0$
 $\Rightarrow x_1 = \frac{-7 + \sqrt{25}}{2 \cdot 2} = \frac{-7+5}{2 \cdot 2} = \underline{-\frac{1}{2}}$ et $x_2 = \frac{-7 - \sqrt{25}}{2 \cdot 2} = \frac{-7-5}{4} = \underline{-3}.$

$$11. 3x^2 - 10x + 3 = 0: a=3, b=-10, c=3, b^2 - 4ac = (-10)^2 - 4 \cdot 3 \cdot 3 = 100 - 36 = 64 > 0$$

$$\Rightarrow x_1 = \frac{10 + \sqrt{64}}{2 \cdot 3} = \frac{10 + 8}{6} = \underline{3} \text{ et } x_2 = \frac{10 - \sqrt{64}}{2 \cdot 3} = \frac{10 - 8}{6} = \underline{\frac{1}{3}}$$

$$12. 5x^2 + 2x = 0: a=5, b=2, c=0, b^2 - 4ac = 2^2 - 4 \cdot 5 \cdot 0 = 4 > 0$$

$$\Rightarrow x_1 = \frac{-2 + \sqrt{4}}{2 \cdot 5} = \frac{-2 + 2}{10} = \underline{0} \text{ et } x_2 = \frac{-2 - \sqrt{4}}{2 \cdot 5} = \frac{-2 - 2}{10} = \underline{-\frac{2}{5}}$$

$$13. x^2 = 5x + 24 \Rightarrow x^2 - 5x - 24 = 0: a=1, b=-5, c=-24, b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot (-24) = 25 + 96 =$$

$$= 121 > 0 \Rightarrow x_1 = \frac{5 + \sqrt{121}}{2} = \frac{5 + 11}{2} = \underline{8} \text{ et } x_2 = \frac{5 - \sqrt{121}}{2} = \frac{5 - 11}{2} = \underline{-3}$$

$$14. 2x^2 + 3x - 2 = 0: a=2, b=3, c=-2, b^2 - 4ac = 3^2 - 4 \cdot 2 \cdot (-2) = 9 + 16 = 25 > 0$$

$$\Rightarrow x_1 = \frac{-3 + \sqrt{25}}{2 \cdot 2} = \frac{-3 + 5}{4} = \underline{\frac{1}{2}} \text{ et } x_2 = \frac{-3 - \sqrt{25}}{2 \cdot 2} = \frac{-3 - 5}{4} = \underline{-2}$$

$$15. 4x^2 - 4x - 8 = 0: a=4, b=-4, c=-8, b^2 - 4ac = (-4)^2 - 4 \cdot 4 \cdot (-8) = 16 + 128 = 144 > 0$$

$$\Rightarrow x_1 = \frac{4 + \sqrt{144}}{2 \cdot 4} = \frac{4 + 12}{8} = \underline{2} \text{ et } x_2 = \frac{4 - \sqrt{144}}{2 \cdot 4} = \frac{4 - 12}{8} = \underline{-1}$$

$$16. 3x^2 - 22x + 35 = 0: a=3, b=-22, c=35, b^2 - 4ac = (-22)^2 - 4 \cdot 3 \cdot 35 = 484 - 420 = 64 > 0$$

$$\Rightarrow x_1 = \frac{22 + \sqrt{64}}{2 \cdot 3} = \frac{22 + 8}{6} = \underline{5} \text{ et } x_2 = \frac{22 - \sqrt{64}}{2 \cdot 3} = \frac{22 - 8}{6} = \underline{\frac{7}{3}}$$

Exercice 2

$$1. \frac{1}{4}x^2 - x - 8 = 0 \xrightarrow{\cdot 4} x^2 - 4x - 32 = 0 : a=1, b=-4, c=-32, b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot (-32) = 16 + 128 = 144 > 0 \Rightarrow x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{4 + \sqrt{144}}{2 \cdot 1} = \frac{4 + 12}{2} = 8 \text{ et}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{4 - \sqrt{144}}{2 \cdot 1} = \frac{4 - 12}{2} = -4.$$

$$2. 2x^2 + 9x + 7 = 0 : a=2, b=9, c=7, b^2 - 4ac = 9^2 - 4 \cdot 2 \cdot 7 = 81 - 56 = 25 > 0$$

$$\Rightarrow x_1 = \frac{-9 + \sqrt{25}}{2 \cdot 2} = \frac{-9 + 5}{4} = -1 \text{ et } x_2 = \frac{-9 - \sqrt{25}}{2 \cdot 2} = \frac{-9 - 5}{4} = -\frac{7}{2}.$$

$$3. 4x^2 + x - 5 = 0 : a=4, b=1, c=-5, b^2 - 4ac = 1^2 - 4 \cdot 4 \cdot (-5) = 1 + 80 = 81 > 0$$

$$\Rightarrow x_1 = \frac{-1 + \sqrt{81}}{2 \cdot 4} = \frac{-1 + 9}{8} = 1 \text{ et } x_2 = \frac{-1 - \sqrt{81}}{2 \cdot 4} = \frac{-1 - 9}{8} = -\frac{5}{4}.$$

$$4. 9x^2 - 6x + 1 = 0 : a=9, b=-6, c=1, b^2 - 4ac = (-6)^2 - 4 \cdot 9 \cdot 1 = 36 - 36 = 0$$

$$\Rightarrow x = \frac{6}{2 \cdot 9} = \frac{1}{3} \text{ solution unique.}$$

$$5. x^2 + x + 1 = 0 : a=1, b=1, c=1, b^2 - 4ac = 1^2 - 4 \cdot 1 \cdot 1 = 1 - 4 = -3 < 0 \Rightarrow \text{pas de solution.}$$

$$6. \frac{3}{4}x^2 + 2x + \frac{4}{3} = 0 \xrightarrow{\cdot 12} 9x^2 + 24x + 16 = 0 : a=9, b=24, c=16, b^2 - 4ac = 24^2 - 4 \cdot 9 \cdot 16 = 576 - 576 = 0$$

$$\Rightarrow x = \frac{-24}{2 \cdot 9} = -\frac{4}{3} \text{ solution unique } (x = -\frac{4}{3})$$

$$7. 12x^2 - 34x - 6 = 0 : a=12, b=-34, c=-6, b^2 - 4ac = (-34)^2 - 4 \cdot 12 \cdot (-6) = 1156 + 288 = 1444 > 0$$

$$\Rightarrow x_1 = \frac{34 + \sqrt{1444}}{2 \cdot 12} = \frac{34 + 38}{24} = 3 \text{ et}$$

$$x_2 = \frac{34 - \sqrt{1444}}{2 \cdot 12} = \frac{34 - 38}{24} = -\frac{1}{6}.$$

$$8. x^2 - 6x + 10 = 0 : a=1, b=-6, c=10, b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 10 = 36 - 40 = -4 < 0$$

$$\Rightarrow \text{pas de solution.}$$

$$9. x^2 - 2x - 3 = 0 : a=1, b=-2, c=-3, b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot (-3) = 4 + 12 = 16 > 0$$

$$\Rightarrow x_1 = \frac{2 + \sqrt{16}}{2} = \frac{2 + 4}{2} = 3 \text{ et } x_2 = \frac{2 - \sqrt{16}}{2} = \frac{2 - 4}{2} = -1.$$

$$10. 4x^2 - 12x + 9 = 0 : a=4, b=-12, c=9, b^2 - 4ac = (-12)^2 - 4 \cdot 4 \cdot 9 = 144 - 144 = 0$$

$$\Rightarrow x = \frac{12}{2 \cdot 4} = \frac{3}{2} \text{ solution unique.}$$

$$11. \frac{1}{6}x^2 + \frac{2}{3}x - 10 = 0 \xrightarrow{\cdot 6} x^2 + 4x - 60 = 0 : a=1, b=4, c=-60, b^2 - 4ac = 4^2 + 4 \cdot 1 \cdot (-60) = 16 + 240 = 256 > 0$$

$$\Rightarrow x_1 = \frac{-4 + \sqrt{256}}{2 \cdot 1} = \frac{-4 + 16}{2} = 6 \text{ et}$$

$$x_2 = \frac{-4 - \sqrt{256}}{2 \cdot 1} = \frac{-4 - 16}{2} = -10.$$

$$12. x^2 + 6x - 16 = 0 : a=1, b=6, c=-16, b^2 - 4ac = 6^2 - 4 \cdot 1 \cdot (-16) = 36 + 64 = 100 > 0$$

$$\Rightarrow x_1 = \frac{-6 + \sqrt{100}}{2} = \frac{-6 + 10}{2} = 2 \text{ et } x_2 = \frac{-6 - \sqrt{100}}{2} = \frac{-6 - 10}{2} = -8.$$

$$13. 2x^2 - 3x + 1 = 0: a=2, b=-3, c=1, b^2 - 4ac = (-3)^2 - 4 \cdot 2 \cdot 1 = 9 - 8 = 1 > 0$$

$$\Rightarrow x_1 = \frac{3 + \sqrt{1}}{2 \cdot 2} = \frac{3+1}{4} = 1 \text{ et } x_2 = \frac{3 - \sqrt{1}}{2 \cdot 2} = \frac{3-1}{4} = \frac{1}{2}.$$

$$14. 2x^2 - 13x + 15 = 0: a=2, b=-13, c=15, b^2 - 4ac = (-13)^2 - 4 \cdot 2 \cdot 15 = 169 - 120 = 49 > 0$$

$$\Rightarrow x_1 = \frac{13 + \sqrt{49}}{2 \cdot 2} = \frac{13+7}{4} = 5 \text{ et } x_2 = \frac{13 - \sqrt{49}}{2 \cdot 2} = \frac{13-7}{4} = \frac{3}{2}.$$

$$15. 2x^2 - 3x + 2 = 0: a=2, b=-3, c=2, b^2 - 4ac = (-3)^2 - 4 \cdot 2 \cdot 2 = 9 - 16 = -7 < 0 \Rightarrow \text{pas de solution.}$$

$$16. x^2 - \frac{3}{4}x + \frac{1}{8} = 0 \stackrel{\cdot 8}{=} 8x^2 - 6x + 1 = 0: a=8, b=-6, c=1, b^2 - 4ac = (-6)^2 - 4 \cdot 8 \cdot 1 = 36 - 32 =$$

$$= 4 > 0 \Rightarrow x_1 = \frac{6 + \sqrt{4}}{2 \cdot 8} = \frac{6+2}{16} = \frac{1}{2} \text{ et } x_2 = \frac{6 - \sqrt{4}}{2 \cdot 8} = \frac{6-2}{16} = \frac{1}{4}.$$

$$17. 5x^2 + 24x - 5 = 0: a=5, b=24, c=-5, b^2 - 4ac = 24^2 - 4 \cdot 5 \cdot (-5) = 576 + 100 = 676 > 0$$

$$\Rightarrow x_1 = \frac{-24 + \sqrt{676}}{2 \cdot 5} = \frac{-24 + 26}{10} = \frac{1}{5} \text{ et } x_2 = \frac{-24 - 26}{2 \cdot 5} = \frac{-50}{10} = -5.$$

$$18. -2x^2 - 4x + 6 = 0: a=-2, b=-4, c=6, b^2 - 4ac = (-4)^2 - 4 \cdot (-2) \cdot 6 = 16 + 48 = 64 > 0$$

$$\Rightarrow x_1 = \frac{4 + \sqrt{64}}{2 \cdot (-2)} = \frac{4+8}{-4} = -3 \text{ et } x_2 = \frac{4 - \sqrt{64}}{2 \cdot (-2)} = \frac{4-8}{-4} = 1.$$

$$19. x^2 + 3x + 4 = 0: a=1, b=3, c=4, b^2 - 4ac = 3^2 - 4 \cdot 1 \cdot 4 = 9 - 16 = -7 < 0 \Rightarrow \text{pas de solution.}$$

$$20. -2x^2 + 2x - 1 = 0: a=-2, b=2, c=-1, b^2 - 4ac = 2^2 - 4 \cdot (-2) \cdot (-1) = 4 - 8 = -4 < 0$$

$$\Rightarrow \text{pas de solution.}$$

Exercice 3

$$\begin{aligned}
 1. \quad & \frac{1-8x}{2} - \frac{x^2-7}{4} + 2x = 0 \\
 & \frac{2-16x}{4} - \frac{(x^2-7)}{4} + \frac{8x}{4} = 0 \\
 & 2-16x - (x^2-7) + 8x = 0 \\
 & 2-16x - x^2 + 7 + 8x = 0 \\
 & -x^2 - 8x + 9 = 0 \\
 & x^2 + 8x - 9 = 0
 \end{aligned}$$

D.C.
 .4
 D
 R
 .(-1)

$$a=1, b=8, c=-9, b^2-4ac = 8^2 - 4 \cdot 1 \cdot (-9) = 64 + 36 = 100 > 0$$

$$\Rightarrow x_1 = \frac{-b + \sqrt{b^2-4ac}}{2a} = \frac{-8 + \sqrt{100}}{2 \cdot 1} = \frac{-8+10}{2} = \underline{1} \text{ et}$$

$$x_2 = \frac{-b - \sqrt{b^2-4ac}}{2a} = \frac{-8 - \sqrt{100}}{2 \cdot 1} = \frac{-8-10}{2} = \underline{-9}$$

$$\begin{aligned}
 2. \quad & \frac{5-4x}{2} + \frac{3x^2-1}{3} = \frac{2x^2+5}{6} \\
 & \frac{15-12x}{6} + \frac{6x^2-2}{6} = \frac{2x^2+5}{6} \\
 & 15-12x+6x^2-2 = 2x^2+5 \\
 & 6x^2-12x+13 = 2x^2+5 \\
 & 4x^2-12x+8 = 0 \\
 & x^2-3x+2 = 0
 \end{aligned}$$

D.C.
 .6
 R
 -2x^2-5
 :4

$$a=1, b=-3, c=2, b^2-4ac = (-3)^2 - 4 \cdot 1 \cdot 2 = 9 - 8 = 1 > 0$$

$$\Rightarrow x_1 = \frac{3 + \sqrt{1}}{2 \cdot 1} = \frac{3+1}{2} = \underline{2} \text{ et } x_2 = \frac{3 - \sqrt{1}}{2 \cdot 1} = \frac{3-1}{2} = \underline{1}$$

$$\begin{aligned}
 3. \quad & 3x^2 - 2x + \frac{1}{3} = 0 \xrightarrow{\cdot 3} 9x^2 - 6x + 1 = 0 : a=9, b=-6, c=1, b^2-4ac = (-6)^2 - 4 \cdot 9 \cdot 1 = \\
 & = 36 - 36 = 0 \Rightarrow x = -\frac{b}{2a} = -\frac{-6}{2 \cdot 9} = \frac{6}{18} = \underline{\frac{1}{3}} \text{ seule solution.}
 \end{aligned}$$

$$4. \quad \frac{x}{x-2} + \frac{1}{2} = \frac{x+6}{x}$$

$$\frac{2x^2}{2x(x-2)} + \frac{x(x-2)}{2x(x-2)} = \frac{2(x+6)(x-2)}{2x(x-2)}$$

$$2x^2 + x(x-2) = 2(x+6)(x-2)$$

$$2x^2 + x^2 - 2x = (2x+12)(x-2)$$

$$3x^2 - 2x = 2x^2 - 4x + 12x - 24$$

$$3x^2 - 2x = 2x^2 + 8x - 24$$

$$x^2 - 10x + 24 = 0$$

$$a=1, b=-10, c=24, b^2-4ac = (-10)^2 - 4 \cdot 1 \cdot 24 = 100 - 96 = 4 > 0$$

$$\Rightarrow x_1 = \frac{10 + \sqrt{4}}{2} = \frac{10+2}{2} = \underline{6} \text{ et } x_2 = \frac{10 - \sqrt{4}}{2} = \frac{10-2}{2} = \underline{4}$$

D.C. (on doit avoir $x \neq 0$ et $x \neq 2$ sinon on divise
 -2x(x-2) par 0)

D

R+D

R

$$-2x^2 - 8x + 24$$

5. $(2x+0,2)^2 - (3x-0,7)^2 = 0$
 $4x^2 + 0,8x + 0,04 - (9x^2 - 4,2x + 0,49) = 0$
 $4x^2 + 0,8x + 0,04 - 9x^2 + 4,2x - 0,49 = 0$
 $-5x^2 + 5x - 0,45 = 0$
 $x^2 - x + 0,09 = 0$

Identité remarquable ou distributivité
)
 R
 : (-5)

$a=1, b=-1, c=0,09, b^2-4ac = (-1)^2 - 4 \cdot 1 \cdot 0,09 = 1 - 0,36 = 0,64 > 0$
 $\Rightarrow x_1 = \frac{1 + \sqrt{0,64}}{2} = \frac{1 + 0,8}{2} = \underline{0,9}$ et $x_2 = \frac{1 - \sqrt{0,64}}{2} = \frac{1 - 0,8}{2} = \underline{0,1}$

6. $(5x-2)(x+1) = 5(x+0,6) - 5$
 $5x^2 + 5x - 2x - 2 = 5x + 3 - 5$
 $5x^2 + 3x - 2 = 5x - 2$
 $5x^2 - 2x = 0$

)
 R
 -5x+2

$a=5, b=-2, c=0, b^2-4ac = (-2)^2 - 4 \cdot 5 \cdot 0 = 4 > 0$
 $\Rightarrow x_1 = \frac{2 + \sqrt{4}}{2 \cdot 5} = \frac{2+2}{10} = \underline{\frac{2}{5}}$ et $x_2 = \frac{2 - \sqrt{4}}{2 \cdot 5} = \frac{2-2}{10} = \underline{0}$

7. $\frac{x^2}{8} = (x-1)(x+1) + \frac{1}{8}$
 $\frac{x^2}{8} = x^2 - 1 + \frac{1}{8}$
 $x^2 = 8x^2 - 8 + 1$
 $x^2 = 8x^2 - 7$
 $0 = 7x^2 - 7$
 $x^2 - 1 = 0$

Identité remarquable ou distributivité
 -8
 R
 -x^2
 :7

$a=1, b=0, c=-1, b^2-4ac = 0^2 - 4 \cdot 1 \cdot (-1) = 4 > 0$
 $\Rightarrow x_1 = \frac{0 + \sqrt{4}}{2} = \frac{2}{2} = \underline{1}$ et $x_2 = \frac{0 - \sqrt{4}}{2} = \frac{-2}{2} = \underline{-1}$

8. $x^2 - 4\sqrt{2}x + 6 = 0: a=1, b=-4\sqrt{2}, c=6, b^2-4ac = (-4\sqrt{2})^2 - 4 \cdot 1 \cdot 6 =$
 $= 16 \cdot 2 - 24 = 32 - 24 = 8 > 0, \sqrt{b^2-4ac} = \sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$
 $\Rightarrow x_1 = \frac{4\sqrt{2} + 2\sqrt{2}}{2} = \frac{6\sqrt{2}}{2} = \underline{3\sqrt{2}} \approx 4,24$ et
 $x_2 = \frac{4\sqrt{2} - 2\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \underline{\sqrt{2}} \approx 1,41$

Exercice 4

1. $x^4 - 5x^2 + 4 = 0$: on pose $u = x^2$; on a $u^2 = (x^2)^2 = x^4$;

l'équation s'écrit $u^2 - 5u + 4 = 0$;

on a $a=1$, $b=-5$, $c=4$, $b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 4 = 25 - 16 = 9 > 0$

$\Rightarrow u_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{5 + \sqrt{9}}{2 \cdot 1} = \frac{5+3}{2} = 4$ et

$u_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{5 - \sqrt{9}}{2 \cdot 1} = \frac{5-3}{2} = 1$;

Comme $u = x^2$, on doit finalement résoudre $x^2 = 4$ et $x^2 = 1$:

$x^2 = 4 \Rightarrow \underline{x = 2}$ et $\underline{x = -2}$;

$x^2 = 1 \Rightarrow \underline{x = 1}$ et $\underline{x = -1}$.

2. $x^4 - 29x^2 + 100 = 0$: on pose $u = x^2$; on a $u^2 = (x^2)^2 = x^4$;

l'équation s'écrit $u^2 - 29u + 100 = 0$;

on a $a=1$, $b=-29$, $c=100$, $b^2 - 4ac = (-29)^2 - 4 \cdot 1 \cdot 100 = 841 - 400 =$

$= 441 > 0 \Rightarrow u_1 = \frac{29 + \sqrt{441}}{2} = \frac{29 + 21}{2} = 25$ et

$u_2 = \frac{29 - \sqrt{441}}{2} = \frac{29 - 21}{2} = 4$;

Comme $u = x^2$, on doit finalement résoudre $x^2 = 25$ et $x^2 = 4$:

$x^2 = 25 \Rightarrow \underline{x = 5}$ et $\underline{x = -5}$;

$x^2 = 4 \Rightarrow \underline{x = 2}$ et $\underline{x = -2}$.

3. $x^6 - 7x^3 - 8 = 0$: on pose $u = x^3$; on a $u^2 = (x^3)^2 = x^6$;

l'équation s'écrit $u^2 - 7u - 8 = 0$;

on a $a=1$, $b=-7$, $c=-8$, $b^2 - 4ac = (-7)^2 - 4 \cdot 1 \cdot (-8) = 49 + 32 = 81 > 0$

$\Rightarrow u_1 = \frac{7 + \sqrt{81}}{2} = \frac{7+9}{2} = 8$ et $u_2 = \frac{7 - \sqrt{81}}{2} = \frac{7-9}{2} = -1$;

Comme $u = x^3$, on doit finalement résoudre $x^3 = 8$ et $x^3 = -1$:

$x^3 = 8 \Rightarrow x = \sqrt[3]{8} = \underline{2}$;

$x^3 = -1 \Rightarrow x = \sqrt[3]{-1} = \underline{-1}$.

4. $x^4 - 5x^2 - 36 = 0$: on pose $u = x^2$; on a $u^2 = (x^2)^2 = x^4$;

l'équation s'écrit $u^2 - 5u - 36 = 0$;

on a $a=1$, $b=-5$, $c=-36$, $b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot (-36) = 25 + 144 = 169 > 0$

$\Rightarrow u_1 = \frac{5 + \sqrt{169}}{2} = \frac{5+13}{2} = 9$ et $u_2 = \frac{5 - \sqrt{169}}{2} = \frac{5-13}{2} = -4$;

Comme $u = x^2$, on doit finalement résoudre $x^2 = 9$ et $x^2 = -4$:

$x^2 = 9 \Rightarrow \underline{x = 3}$ et $\underline{x = -3}$;

$x^2 = -4$ impossible.

5. $x^{10} + 31x^5 - 32 = 0$: on pose $u = x^5$; on a $u^2 = (x^5)^2 = x^{10}$;
 l'équation s'écrit $u^2 + 31u - 32 = 0$;
 on a $a = 1, b = 31, c = -32, b^2 - 4ac = 31^2 - 4 \cdot 1 \cdot (-32) = 961 + 128 = 1089 > 0$
 $\Rightarrow u_1 = \frac{-31 + \sqrt{1089}}{2} = \frac{-31 + 33}{2} = 1$ et
 $u_2 = \frac{-31 - \sqrt{1089}}{2} = \frac{-31 - 33}{2} = -32$;

Comme $u = x^5$, on doit finalement résoudre $x^5 = 1$ et $x^5 = -32$:
 $x^5 = 1 \Rightarrow x = \sqrt[5]{1} = 1$;
 $x^5 = -32 \Rightarrow x = \sqrt[5]{-32} = -2$.

6. $x^8 - 626x^4 + 625 = 0$: on pose $u = x^4$, on a $u^2 = (x^4)^2 = x^8$;
 l'équation s'écrit $u^2 - 626u + 625 = 0$;
 on a $a = 1, b = -626, c = 625, b^2 - 4ac = (-626)^2 - 4 \cdot 1 \cdot 625 = 391876 - 2500 = 389376 > 0$
 $\Rightarrow u_1 = \frac{626 + \sqrt{389376}}{2} = \frac{626 + 624}{2} = 625$ et
 $u_2 = \frac{626 - \sqrt{389376}}{2} = \frac{626 - 624}{2} = 1$;

Comme $u = x^4$, on doit finalement résoudre $x^4 = 625$ et $x^4 = 1$:
 $x^4 = 625 \Rightarrow x = \pm \sqrt[4]{625} \Rightarrow x = 5$ et $x = -5$;
 $x^4 = 1 \Rightarrow x = \pm \sqrt[4]{1} \Rightarrow x = 1$ et $x = -1$.

7. $36x^4 - 65x^2 - 36 = 0$: on pose $u = x^2$, on a $u^2 = (x^2)^2 = x^4$;
 l'équation s'écrit $36u^2 - 65u - 36 = 0$;
 on a $a = 36, b = -65, c = -36, b^2 - 4ac = (-65)^2 - 4 \cdot 36 \cdot (-36) = 4225 + 5184 = 9409 > 0$
 $\Rightarrow u_1 = \frac{65 + \sqrt{9409}}{2 \cdot 36} = \frac{65 + 97}{72} = \frac{9}{4}$ et $u_2 = \frac{65 - \sqrt{9409}}{2 \cdot 36} = \frac{65 - 97}{72} = -\frac{4}{9}$;
 Comme $u = x^2$, on doit finalement résoudre $x^2 = \frac{9}{4}$ et $x^2 = -\frac{4}{9}$:
 $x^2 = \frac{9}{4} \Rightarrow x = \frac{3}{2}$ et $x = -\frac{3}{2}$;
 $x^2 = -\frac{4}{9}$ impossible.

8. $4x^4 - 73x^2 + 144 = 0$: on pose $u = x^2$; on a $u^2 = (x^2)^2 = x^4$;
 l'équation s'écrit $4u^2 - 73u + 144 = 0$;
 on a $a = 4, b = -73, c = 144, b^2 - 4ac = (-73)^2 - 4 \cdot 4 \cdot 144 = 3025 > 0$
 $\Rightarrow u_1 = \frac{73 + \sqrt{3025}}{2 \cdot 4} = 16$ et $u_2 = \frac{73 - \sqrt{3025}}{2 \cdot 4} = \frac{9}{4}$;
 Comme $u = x^2$, on doit finalement résoudre $x^2 = 16$ et $x^2 = \frac{9}{4}$:
 $x^2 = 16 \Rightarrow x = 4$ et $x = -4$;
 $x^2 = \frac{9}{4} \Rightarrow x = \frac{3}{2}$ et $x = -\frac{3}{2}$.

Exercice 5

1. $\sqrt{2x-9} = \frac{1}{3}$	$(\quad)^2$
$2x-9 = \frac{1}{9}$	$\cdot 9$
$18x-81 = 1$	$+81$
$18x = 82$	$:18$
$x = \frac{41}{9}$	

Comme on a mis l'équation à la puissance 2, on doit vérifier la solution:

$x = \frac{41}{9} \Rightarrow \sqrt{2x-9} = \sqrt{2 \cdot \frac{41}{9} - 9} = \sqrt{\frac{82}{9} - \frac{81}{9}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$ ok.

La solution est donc $x = \frac{41}{9}$.

2. $\sqrt{3-x} - x = 3$	$+x$
$\sqrt{3-x} = x+3$	$(\quad)^2$
$3-x = (x+3)^2$	identité remarquable ou distributive
$3-x = x^2+6x+9$	$-3+x$
$0 = x^2+7x+6$	

$a=1, b=7, c=6, b^2-4ac = 7^2 - 4 \cdot 1 \cdot 6 = 49 - 24 = 25 > 0$

$\Rightarrow x_1 = \frac{-7+\sqrt{25}}{2} = \frac{-7+5}{2} = -1$ et $x_2 = \frac{-7-\sqrt{25}}{2} = \frac{-7-5}{2} = -6$.

Comme on a mis l'équation à la puissance 2, on doit vérifier les solutions:

$x = -1 \Rightarrow \sqrt{3-x} - x = \sqrt{3-(-1)} - (-1) = \sqrt{4} + 1 = 2 + 1 = 3$ ok ;

$x = -6 \Rightarrow \sqrt{3-(-6)} - (-6) = \sqrt{9} + 6 = 3 + 6 = 9 \neq 3$ Ko.

La solution est donc $x = -1$.

3. $x - \sqrt{x} - 6 = 0$	$+\sqrt{x}$
$x - 6 = \sqrt{x}$	$(\quad)^2$
$(x-6)^2 = x$	identité remarquable ou distributive
$x^2 - 12x + 36 = x$	$-x$
$x^2 - 13x + 36 = 0$	

$a=1, b=-13, c=36, b^2-4ac = (-13)^2 - 4 \cdot 1 \cdot 36 = 169 - 144 = 25 > 0$

$\Rightarrow x_1 = \frac{13+\sqrt{25}}{2} = \frac{13+5}{2} = 9$ et $x_2 = \frac{13-\sqrt{25}}{2} = \frac{13-5}{2} = 4$.

Comme on a mis l'équation à la puissance 2, on doit vérifier les solutions:

$x = 9 \Rightarrow x - \sqrt{x} - 6 = 9 - \sqrt{9} - 6 = 9 - 3 - 6 = 0$ ok

$x = 4 \Rightarrow x - \sqrt{x} - 6 = 4 - \sqrt{4} - 6 = 4 - 2 - 6 = -4 \neq 0$ Ko.

La solution est donc $x = 9$.

$$\begin{array}{l|l}
 4. \quad \sqrt{7-x} = x-5 & ()^2 \\
 7-x = (x-5)^2 & \text{identité remarquable ou distributivité} \\
 7-x = x^2 - 10x + 25 & -7+x \\
 0 = x^2 - 9x + 18 &
 \end{array}$$

$a=1, b=-9, c=18, b^2-4ac = (-9)^2 - 4 \cdot 1 \cdot 18 = 81 - 72 = 9 > 0$
 $\Rightarrow x_1 = \frac{9+\sqrt{9}}{2} = \frac{9+3}{2} = 6$ et $x_2 = \frac{9-\sqrt{9}}{2} = \frac{9-3}{2} = 3$;

Comme on a mis l'équation à la puissance 2, on doit vérifier les solutions:

$x=6 \Rightarrow \sqrt{7-x} = \sqrt{7-6} = \sqrt{1} = 1$ et $x-5 = 6-5 = 1$ OK

$x=3 \Rightarrow \sqrt{7-x} = \sqrt{7-3} = \sqrt{4} = 2$ et $x-5 = 3-5 = -2$ KO.

La solution est donc $x=6$.

$$\begin{array}{l|l}
 5. \quad \sqrt{x^2+7} = 4 & ()^2 \\
 x^2+7 = 16 & -7 \\
 x^2 = 9 & \sqrt{\quad} \\
 x=3 \text{ ou } x=-3 &
 \end{array}$$

Comme on a mis l'équation à la puissance 2, on doit vérifier les solutions:

$x=3 \Rightarrow \sqrt{x^2+7} = \sqrt{3^2+7} = \sqrt{16} = 4$ OK

$x=-3 \Rightarrow \sqrt{x^2+7} = \sqrt{(-3)^2+7} = \sqrt{16} = 4$ OK.

Les solutions sont donc $x=3$ et $x=-3$.

$$\begin{array}{l|l}
 6. \quad x + \sqrt{5x+10} = 8 & -x \\
 \sqrt{5x+10} = 8-x & ()^2 \\
 5x+10 = (8-x)^2 & \text{identité remarquable ou distributivité} \\
 5x+10 = 64 - 16x + x^2 & -5x - 10 \\
 0 = x^2 - 21x + 54 &
 \end{array}$$

$a=1, b=-21, c=54, b^2-4ac = (-21)^2 - 4 \cdot 1 \cdot 54 = 441 - 216 = 225 > 0$
 $\Rightarrow x_1 = \frac{21+\sqrt{225}}{2} = \frac{21+15}{2} = 18$ et $x_2 = \frac{21-\sqrt{225}}{2} = \frac{21-15}{2} = 3$;

Comme on a mis l'équation à la puissance 2, on doit vérifier les solutions:

$x=18 \Rightarrow x + \sqrt{5x+10} = 18 + \sqrt{5 \cdot 18 + 10} = 18 + \sqrt{100} = 18 + 10 = 28 \neq 8$ KO

$x=3 \Rightarrow x + \sqrt{5x+10} = 3 + \sqrt{5 \cdot 3 + 10} = 3 + \sqrt{25} = 3 + 5 = 8$ OK.

La solution est $x=3$.

7. $\sqrt[3]{x^3-56} = x-2$

$x^3-56 = (x-2)^3$

$x^3-56 = x^3-6x^2+12x-8$

$0 = -6x^2+12x+48$

$0 = x^2-2x-8$

$a=1, b=-2, c=-8, b^2-4ac = (-2)^2-4 \cdot 1 \cdot (-8) = 4+32 = 36 > 0$

$\Rightarrow x_1 = \frac{2+\sqrt{36}}{2} = \frac{2+6}{2} = 4$ et $x_2 = \frac{2-\sqrt{36}}{2} = \frac{2-6}{2} = -2$;

Comme on a mis l'Equation à la puissance 3, on doit vérifier les solutions:

$x=4 \Rightarrow \sqrt[3]{x^3-56} = \sqrt[3]{4^3-56} = \sqrt[3]{64-56} = \sqrt[3]{8} = 2$ et $x-2 = 4-2 = 2$ ok

$x=-2 \Rightarrow \sqrt[3]{x^3-56} = \sqrt[3]{-8-56} = \sqrt[3]{-64} = -4$ et $x-2 = -2-2 = -4$ ok.

Les solutions sont donc $x=4$ et $x=-2$.

$()^3$

identité remarquable: $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

$-x^3+56$

$: (-6)$

8. $x^2-2\sqrt{x^2-9} = 17$

$x^2-17 = 2\sqrt{x^2-9}$

$(x^2-17)^2 = 4(x^2-9)$

$x^4-34x^2+289 = 4x^2-36$

$x^4-38x^2+325 = 0$

$+2\sqrt{x^2-9} - 17$

$()^2$

identité remarquable et distributivité

$-4x^2+36$

On pose $u = x^2$; on a $u^2 = (x^2)^2 = x^4$; l'Equation devient $u^2-38u+325 = 0$;

on a $a=1, b=-38, c=325, b^2-4ac = (-38)^2-4 \cdot 1 \cdot 325 = 1444-1300 = 144 > 0$

$\Rightarrow u_1 = \frac{38+\sqrt{144}}{2} = \frac{38+12}{2} = 25$ et $u_2 = \frac{38-\sqrt{144}}{2} = \frac{38-12}{2} = 13$;

Comme $u = x^2$, on doit maintenant résoudre $x^2 = 25$ et $x^2 = 13$:

$x^2 = 25 \Rightarrow x = 5$ et $x = -5$;

$x^2 = 13 \Rightarrow x = \sqrt{13}$ et $x = -\sqrt{13}$;

Comme on a mis l'Equation à la puissance 2, on doit vérifier les solutions:

$x=5 \Rightarrow x^2-2\sqrt{x^2-9} = 5^2-2\sqrt{5^2-9} = 25-2\sqrt{25-9} = 25-2 \cdot 4 = 17$ ok

$x=-5 \Rightarrow x^2-2\sqrt{x^2-9} = (-5)^2-2\sqrt{(-5)^2-9} = 25-2\sqrt{25-9} = 17$ ok

$x=\sqrt{13} \Rightarrow x^2-2\sqrt{x^2-9} = 13-2\sqrt{13-9} = 13-2\sqrt{4} = 13-2 \cdot 2 = 9 \neq 17$ Ko

$x=-\sqrt{13} \Rightarrow x^2-2\sqrt{x^2-9} = 13-2\sqrt{13-9} = 9 \neq 17$ Ko.

Les solutions sont donc $x=5$ et $x=-5$.

$$9. \sqrt{6-x} + 2x = -3$$

$$\sqrt{6-x} = -2x-3$$

$$6-x = (-2x-3)^2$$

$$6-x = 4x^2 + 12x + 9$$

$$0 = 4x^2 + 13x + 3$$

$$-2x$$

$$(\quad)^2$$

identité remarquable ou distributivité

$$-6+x$$

$$a=4, b=13, c=3, b^2-4ac = 13^2 - 4 \cdot 4 \cdot 3 = 169 - 48 = 121 > 0$$

$$\Rightarrow x_1 = \frac{-13 + \sqrt{121}}{8} = \frac{-13 + 11}{8} = -\frac{1}{4} \text{ et } x_2 = \frac{-13 - \sqrt{121}}{8} = \frac{-13 - 11}{8} = -3;$$

Comme on a mis l'équation à la puissance 2, on doit vérifier les solutions:

$$x = -\frac{1}{4} \Rightarrow \sqrt{6-x} + 2x = \sqrt{6 + \frac{1}{4}} + 2 \cdot (-\frac{1}{4}) = \sqrt{\frac{25}{4}} - \frac{1}{2} = \frac{5}{2} - \frac{1}{2} = 2 \neq -3 \text{ KO}$$

$$x = -3 \Rightarrow \sqrt{6-x} + 2x = \sqrt{6+3} + 2 \cdot (-3) = 3 - 6 = -3 \text{ OK.}$$

La solution est donc $x = -3$.

Exercice 6

1. $\begin{cases} x+y=4 \\ x^2+y^2=26 \end{cases} \Rightarrow y=4-x$
 $\hookrightarrow x^2+(4-x)^2=26 \Rightarrow x^2+16-8x+x^2=26$
 $\Rightarrow 2x^2-8x+16=26 \Rightarrow 2x^2-8x-10=0 \Rightarrow x^2-4x-5=0$;
 $a=1, b=-4, c=-5, b^2-4ac=(-4)^2-4 \cdot 1 \cdot (-5)=16+20=36 > 0$
 $\Rightarrow x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} = \frac{4+\sqrt{36}}{2 \cdot 1} = \frac{4+6}{2} = 5 \Rightarrow y_1 = 4-x_1 = 4-5 = -1$
 et $x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{4-\sqrt{36}}{2 \cdot 1} = \frac{4-6}{2} = -1 \Rightarrow y_2 = 4-x_2 = 4-(-1) = 5$.

les couples de solutions sont donc $x_1=5, y_1=-1$ et $x_2=-1, y_2=5$.

2. $\begin{cases} x-y=-11 \\ x^2-y^2=-77 \end{cases} \Rightarrow x=y-11$
 $\hookrightarrow (y-11)^2-y^2=-77 \Rightarrow y^2-22y+121-y^2=-77$
 $\Rightarrow -22y+121=-77 \Rightarrow -22y=-198 \Rightarrow y=9 \Rightarrow x=y-11=9-11=-2$.

le couple de solutions est donc $x=-2$ et $y=9$.

3. $\begin{cases} x^2+y=98 \\ x^2+y^2=104 \end{cases} \Rightarrow x^2=98-y$
 $\hookrightarrow 98-y+y^2=104 \Rightarrow y^2-y-6=0$;
 $a=1, b=-1, c=-6, b^2-4ac=(-1)^2-4 \cdot 1 \cdot (-6)=1+24=25 > 0$
 $\Rightarrow y_1 = \frac{1+\sqrt{25}}{2} = \frac{1+5}{2} = 3 \Rightarrow x^2=98-3=95 \Rightarrow x = \pm\sqrt{95}$
 et $y_2 = \frac{1-\sqrt{25}}{2} = \frac{1-5}{2} = -2 \Rightarrow x^2=98-(-2)=100 \Rightarrow x = \pm 10$.

les couples de solutions sont donc $y_1=3, x_1=\sqrt{95}$ et $y_3=3, x_3=-\sqrt{95}$
 $y_2=-2, x_2=10$ et $y_4=-2, x_4=-10$

4. $\begin{cases} x-y=7 \\ xy=30 \end{cases} \Rightarrow x=y+7$
 $\hookrightarrow (y+7) \cdot y=30 \Rightarrow y^2+7y=30 \Rightarrow y^2+7y-30=0$;
 $a=1, b=7, c=-30, b^2-4ac=7^2-4 \cdot 1 \cdot (-30)=49+120=169 > 0$
 $\Rightarrow y_1 = \frac{-7+\sqrt{169}}{2} = \frac{-7+13}{2} = 3 \Rightarrow x_1 = y_1+7 = 3+7 = 10$
 et $y_2 = \frac{-7-\sqrt{169}}{2} = \frac{-7-13}{2} = -10 \Rightarrow x_2 = y_2+7 = -10+7 = -3$.

les couples de solutions sont donc $x_1=10, y_1=3$ et $x_2=-3, y_2=-10$.

5. $\begin{cases} x^2+y^2=100 \\ xy=48 \end{cases} \Rightarrow y = \frac{48}{x}$
 $\hookrightarrow x^2 + \left(\frac{48}{x}\right)^2 = 100 \Rightarrow x^2 + \frac{2304}{x^2} = 100$
 $\Rightarrow x^4 + 2304 = 100x^2 \Rightarrow x^4 - 100x^2 + 2304 = 0$;
 on pose $u = x^2$ et on a $u^2 = (x^2)^2 = x^4$; l'équation s'écrit alors $u^2 - 100u + 2304 = 0$;
 $a=1, b=-100, c=2304, b^2-4ac=(-100)^2-4 \cdot 1 \cdot 2304 = 10000-9216 = 784 > 0$
 $\Rightarrow u_1 = \frac{100+\sqrt{784}}{2} = 64$ et $u_2 = \frac{100-\sqrt{784}}{2} = 36$;
 comme $u = x^2$, on doit maintenant résoudre $x^2 = 64$ et $x^2 = 36$;

$$x^2 = 64 \Rightarrow \begin{cases} x_1 = 8 \Rightarrow y_1 = \frac{48}{x_1} = \frac{48}{8} = 6 ; \\ x_2 = -8 \Rightarrow y_2 = \frac{48}{x_2} = \frac{48}{-8} = -6 ; \end{cases}$$

$$x^2 = 36 \Rightarrow \begin{cases} x_3 = 6 \Rightarrow y_3 = \frac{48}{x_3} = \frac{48}{6} = 8 ; \\ x_4 = -6 \Rightarrow y_4 = \frac{48}{x_4} = \frac{48}{-6} = -8. \end{cases}$$

Les couples de solutions sont donc $\begin{cases} x_1 = 8 \\ y_1 = 6 \end{cases} \begin{cases} x_2 = -8 \\ y_2 = -6 \end{cases} \begin{cases} x_3 = 6 \\ y_3 = 8 \end{cases} \begin{cases} x_4 = -6 \\ y_4 = -8. \end{cases}$

6. $\begin{cases} x+y = 12 \Rightarrow y = 12-x \\ x^2+xy+y^2 = 109 \end{cases} \rightarrow x^2+x(12-x)+(12-x)^2 = 109$
 $\Rightarrow x^2+12x-x^2+144-24x+x^2 = 109$
 $\Rightarrow x^2-12x+144 = 109 \Rightarrow x^2-12x+35 = 0 ;$

$a=1, b=-12, c=35, b^2-4ac = (-12)^2 - 4 \cdot 1 \cdot 35 = 144 - 140 = 4 > 0$

$\Rightarrow x_1 = \frac{12+\sqrt{4}}{2} = \frac{12+2}{2} = 7 \Rightarrow y_1 = 12-x_1 = 12-7 = 5$

et $x_2 = \frac{12-\sqrt{4}}{2} = \frac{12-2}{2} = 5 \Rightarrow y_2 = 12-x_2 = 12-5 = 7.$

Les couples de solutions sont donc $\underline{x_1=7, y_1=5}$ et $\underline{x_2=5, y_2=7}.$

7. $\begin{cases} x-y = 7 \Rightarrow x = y+7 \\ x^2+xy+y^2 = 139 \end{cases} \rightarrow (y+7)^2 + (y+7)y + y^2 = 139$
 $\Rightarrow y^2+14y+49 + y^2+7y + y^2 = 139$
 $\Rightarrow 3y^2+21y+49 = 139 \Rightarrow 3y^2+21y-90 = 0 ;$

$a=3, b=21, c=-90, b^2-4ac = 21^2 - 4 \cdot 3 \cdot (-90) = 441 + 1080 = 1521 > 0$

$\Rightarrow y_1 = \frac{-21+\sqrt{1521}}{2 \cdot 3} = \frac{-21+39}{6} = 3 \Rightarrow x_1 = y_1+7 = 3+7 = 10$

et $y_2 = \frac{-21-\sqrt{1521}}{2 \cdot 3} = \frac{-21-39}{6} = -10 \Rightarrow x_2 = y_2+7 = -10+7 = -3.$

Les couples de solutions sont donc $\underline{x_1=10, y_1=3}$ et $\underline{x_2=-3, y_2=-10}.$

8. $\begin{cases} 3xy - 4x = 6 \Rightarrow 3xy = 4x+6 \Rightarrow 9xy = 12x+18 \\ x^2+9xy = 63 \end{cases} \rightarrow x^2+12x+18 = 63$
 $\Rightarrow x^2+12x-45 = 0 ; a=1, b=12, c=-45, b^2-4ac = 12^2 - 4 \cdot 1 \cdot (-45) = 324 > 0$
 $\Rightarrow x_1 = \frac{-12+\sqrt{324}}{2} = \frac{-12+18}{2} = 3$ et $x_2 = \frac{-12-\sqrt{324}}{2} = \frac{-12-18}{2} = -15;$

avec $x_1 = 3$, on a $3 \cdot 3y = 4 \cdot 3 + 6 \Rightarrow 9y = 18 \Rightarrow y = 2;$

avec $x_2 = -15$, on a $3 \cdot (-15)y = 4 \cdot (-15) + 6 \Rightarrow -45y = -54 \Rightarrow y = \frac{6}{5}.$

Les couples de solutions sont donc $\underline{x_1=3, y_1=2}$ et $\underline{x_2=-15, y_2=\frac{6}{5}}.$

9.
$$\begin{cases} 2xy - 3y = 3 \Rightarrow 2xy = 3y + 3 \Rightarrow -4xy = -6y - 6 \\ y^2 - 4xy = -15 \end{cases} \Rightarrow y^2 - 6y - 6 = -15 \Rightarrow y^2 - 6y + 9 = 0;$$

$$a=1, b=-6, c=9, b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 9 = 36 - 36 = 0$$

$$\Rightarrow y = -\frac{b}{2a} = \frac{6}{2} = 3 \Rightarrow 2x \cdot 3 = 3 \cdot 3 + 3 \Rightarrow 6x = 12 \Rightarrow x = 2$$

Le couple de solution est donc $x=2, y=3$.

10.
$$\begin{cases} \frac{x}{y} + \frac{y}{x} = -\frac{29}{10} \Rightarrow \frac{x^2}{xy} + \frac{y^2}{xy} = -\frac{29xy}{10xy} \Rightarrow 10x^2 + 10y^2 = -29xy \\ x+y = 3 \Rightarrow y = 3-x \end{cases}$$

$$\Rightarrow 10x^2 + 10(3-x)^2 = -29x(3-x)$$

$$\Rightarrow 10x^2 + 10(9 - 6x + x^2) = -87x + 29x^2$$

$$\Rightarrow 10x^2 + 90 - 60x + 10x^2 = -87x + 29x^2$$

$$\Rightarrow 20x^2 - 60x + 90 = -87x + 29x^2 \Rightarrow -9x^2 + 27x + 90 = 0$$

$$\Rightarrow x^2 - 3x - 10 = 0 ; a=1, b=-3, c=-10, b^2 - 4ac = (-3)^2 + 4 \cdot 1 \cdot (-10) = 49 > 0$$

$$\Rightarrow x_1 = \frac{3 + \sqrt{49}}{2} = \frac{3+7}{2} = 5 \Rightarrow y_1 = 3 - x_1 = 3 - 5 = -2$$

$$\text{et } x_2 = \frac{3 - \sqrt{49}}{2} = \frac{3-7}{2} = -2 \Rightarrow y_2 = 3 - x_2 = 3 - (-2) = 5.$$

Les couples de solution sont donc $x_1=5, y_1=-2$ et $x_2=-2, y_2=5$.

11.
$$\begin{cases} \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{5}{2} \Rightarrow \frac{(x+y)^2 + (x-y)^2}{(x-y)(x+y)} = \frac{5}{2} \Rightarrow \frac{x^2+2xy+y^2+x^2-2xy+y^2}{x^2-y^2} = \frac{5}{2} \\ x^2+y^2=90 \end{cases}$$

$$\Rightarrow \frac{2x^2+2y^2}{x^2-y^2} = \frac{5}{2} \Rightarrow 2(2x^2+2y^2) = 5(x^2-y^2)$$

$$\Rightarrow 4x^2+4y^2 = 5x^2-5y^2 \Rightarrow -x^2 = -9y^2 \Rightarrow x^2 = 9y^2$$

$$\Rightarrow 9y^2+y^2=90 \Rightarrow 10y^2=90 \Rightarrow y^2=9 \Rightarrow y = \pm 3$$

$$\Rightarrow x^2 = 9y^2 = 9 \cdot 9 = 81 \Rightarrow x = \pm 9.$$

Les couples de solutions sont donc $\begin{cases} x_1=9 \\ y_1=3 \end{cases} \quad \begin{cases} x_2=9 \\ y_2=-3 \end{cases} \quad \begin{cases} x_3=-9 \\ y_3=3 \end{cases} \quad \begin{cases} x_4=-9 \\ y_4=-3 \end{cases}$.