Rechercher les intégrales indéfinies suivantes :

 # 1

a) $\int(x^2+3x-5)dx=\frac{x^3}{3}+\frac{3x}{2}-5x+c$	b) $\int\cos(x)dx=\sin(x)+c$
c) $\int 3\cos(2x)dx \underset{u=2x}{=} \frac{3}{2}\sin(2x)+c$	d) $\int 3dx=3x+c$
e) $\int 2e^{3x}dx=\frac{2}{3}e^{3x}+c$	f) $\int\sqrt{x}\cdot dx=\int x^{\frac{1}{2}}\cdot dx=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+c=\frac{2}{3}\sqrt{x^3}+c$
g) $\int dx=x+c$	h) $\int 0=c$
i) $\int\frac{dx}{x}=\ln(x)+c$	l) $\int\frac{dx}{x^2}=\int x^{-2}dx=\frac{x^{-1}}{-1}+c=-\frac{1}{x}+c$

 # 2 Par parties

a) $\int xe^x dx=\int u'v\,dx=uv-\int uv'dx = xe^x-\int e^x dx$
$= xe^x-e^x+c=e^x(x-1)+c$

p.p. $u=e^x \Leftarrow u'=e^x$; $v=x \Rightarrow v'=1$

b) $\int x\cdot\sin(x)\cdot dx=\int u'v\,dx=uv-\int uv'dx$
$=-x\cdot\cos(x)+\int\cos(x)\cdot dx=-x\cdot\cos(x)+\sin(x)+c$

p.p $u=-\cos(x) \Leftarrow u'=\sin(x)$; $v=x \Rightarrow v'=1$

c) $\int x\cos(x)dx=\int u'v\,dx=uv-\int uv'dx$
$= x\cdot\sin(x)-\int\sin(x)\cdot dx= x\cdot\sin(x)-\cos(x)+c$

p.p $u=\sin(x) \Leftarrow u'=\cos(x)$; $v=x \Rightarrow v'=1$

d) $\int x\ln(x)dx=\int u'v\,dx=uv-\int uv'dx=\frac{x^2}{2}\cdot\ln(x)-\frac{1}{2}\int x\,dx$
$=\frac{x^2}{2}\cdot\ln(x)-\frac{x^2}{4}+c=\frac{x^2}{4}(2\ln(x)-1)+c=\frac{x^2}{4}\ln\left(\frac{x^2}{e}\right)+c$

p.p $u=\frac{x^2}{2} \Leftarrow u'=x$; $v=\ln(x) \Rightarrow v'=\frac{1}{x}$

e) $\int x^2\ln(x)dx=\int u'v\,dx=uv-\int uv'dx=\frac{x^3}{3}\cdot\ln(x)-\frac{1}{3}\int x^2\,dx$
$=\frac{x^3}{3}\cdot\ln(x)-\frac{x^3}{9}+c=\frac{x^3}{9}(3\ln(x)-1)+c=\frac{x^3}{9}\ln\left(\frac{x^3}{e}\right)+c$

p.p $u=\frac{x^3}{3} \Leftarrow u'=x^2$; $v=\ln(x) \Rightarrow v'=\frac{1}{x}$

f) $\int x^2e^x dx=\int u'v\,dx=uv-\int uv'dx = x^2e^x-2\underbrace{\int xe^x dx}_{J,\text{cf a)}}$
$=x^2e^x-2(e^x(x-1))+c\ \ e^x(x^2-2x+2)+c$

p.p. $u=e^x \Leftarrow u'=e^x$; $v=x^2 \Rightarrow v'=2x$

g) $\int x^2\cos(x)dx=\int u'v\,dx=uv-\int uv'dx$
$=x^2\cdot\sin(x)-2\underbrace{\int x\cdot\sin(x)\cdot dx}_{J,\text{cf b)}}$
$=x^2\cdot\sin(x)-2(-x\cdot\cos(x)+\sin(x))+c$
$=x^2\cdot\sin(x)+2x\cdot\cos(x)-2\sin(x)+c$

p.p $u=\sin(x) \Leftarrow u'=\cos(x)$; $v=x^2 \Rightarrow v'=2x$

2 Par parties

h) $\int x^2\sin(x)dx = \int u'v\,dx = uv - \int uv'dx$

$= -x^2\cos(x) + 2\underbrace{\int x\cos(x)\cdot dx}_{J,\text{cf c)}}$

$= -x^2\cos(x) + 2\left(x\sin(x) - \cos(x)\right) + c$

$= -x^2\cos(x) + 2x\sin(x) - 2\cos(x) + c$

p.p $u = -\cos(x) \Leftarrow u' = \sin(x)$; $v = x^2 \Rightarrow v' = 2x$

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i) $I = \int \frac{\ln(x)}{x}dx = \int u'v\,dx = uv - \int uv'dx = \ln^2(x) - \underbrace{\int\frac{\ln(x)}{x}dx}_{I}$

$\Rightarrow I = \ln^2(x) - I \Rightarrow I = \frac{\ln^2(x)}{2} + c$

p.p $u = \ln(x) \Leftarrow u' = \frac{1}{x}$; $v = \ln(x) \Rightarrow v' = \frac{1}{x}$

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l) $\int\frac{\ln(x)}{x^2}dx = \int u'v\,dx = uv - \int uv'dx = -\frac{1}{x}\ln(x) + \int\frac{dx}{x^2}$

$= -\frac{1}{x}\ln(x) - \frac{1}{x} + c = -\frac{1}{x}\left(\ln(x)+1\right) + c = -\frac{1}{x}\ln\left(\frac{x}{e}\right) + c$

p.p $u = -\frac{1}{x} \Leftarrow u' = \frac{1}{x^2}$; $v = \ln(x) \Rightarrow v' = \frac{1}{x}$

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k) $I = \int e^x\cos(x)dx = \int u'v\,dx = uv - \int uv'dx$

$= e^x\sin(x) - \underbrace{\int e^x\sin(x)dx}_{J}$

p.p $u = \sin(x) \Leftarrow u' = \cos(x)$; $v = e^x \Rightarrow v' = e^x$

p.p $u = -\cos(x) \Leftarrow u' = \sin(x)$; $v = e^x \Rightarrow v' = e^x$

Calcul de $J = \int e^x\sin(x)dx = \int u'v\,dx = uv - \int uv'dx$

$= -e^x\cos(x) + \underbrace{\int e^x\cos(x)dx}_{I}$

$\Rightarrow I = e^x\sin(x) - \left(-e^x\cos(x) + I\right) \Rightarrow I = \frac{e^x\left(\cos(x)+\sin(x)\right)}{2} + c$

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l) $J = \int e^x\sin(x)dx \underset{\text{cf k)}}{=} e^x\sin(x) - I \Rightarrow J = \frac{e^x\left(\cos(x)-\sin(x)\right)}{2} + c$

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m) $\int\ln(x)dx = \int u'v\,dx = uv - \int uv'dx = x\ln(x) - \int dx$

$= x\ln(x) - x + c = x\left(\ln(x)-1\right) + c$

p.p $u = x \Leftarrow u' = 1$; $v = \ln(x) \Rightarrow v' = \frac{1}{x}$

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n) $I = \int\sin^2(x)dx = \int u'v\,dx = uv - \int uv'dx$

$= -\sin(x)\cos(x) + \int\underbrace{\cos^2(x)}_{1-\sin^2(x)}dx = -\sin(x)\cos(x) + \int dx - \underbrace{\int\sin^2(x)dx}_{I}$

$\Rightarrow I = -\sin(x)\cos(x) + x - I \Rightarrow I = \frac{x - \sin(x)\cos(x)}{2} + c$

p.p $u = -\cos(x) \Leftarrow u' = \sin(x)$; $v = \sin(x) \Rightarrow v' = \cos(x)$

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o) $J = \int\cos^2(x)dx \underset{\text{cf n)}}{=} I + \sin(x)\cos(x) \Rightarrow J = \frac{x + \sin(x)\cos(x)}{2} + c$

3 Par changement de variable

a) $\int \frac{x}{x^2+1}dx = \frac{1}{2}\int \frac{x}{u}\frac{du}{x} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln(u)+c = \underline{\underline{\frac{1}{2}\ln(x^2+1)+c}}$

CV : $u = x^2+1$
$u' = 2x = \frac{du}{dx} \Rightarrow dx = \frac{du}{2x}$

b) $\int x\cdot\sqrt{1+x^2}dx = \frac{1}{2}\int x\sqrt{u}\frac{du}{x} = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\frac{2}{3}\sqrt{u^3}+c = \underline{\underline{\frac{1}{3}\sqrt{1+x^2}+c}}$

CV : $u = x^2+1$
$u' = 2x = \frac{du}{dx} \Rightarrow dx = \frac{du}{2x}$

c) $\int 2x(x^2-5)^8dx = \int 2xu^8\frac{du}{2x} = \int u^8du = \frac{u^9}{9}+c = \underline{\underline{\frac{(x^2-5)^9}{9}+c}}$

CV : $u = x^2-5$
$u' = 2x = \frac{du}{dx} \Rightarrow dx = \frac{du}{2x}$

d) $\int \frac{x^2}{\sqrt{1-x^3}}dx = -\frac{1}{3}\int \frac{x^2}{\sqrt{u}}\frac{du}{x^2} = -\frac{1}{3}\int \frac{du}{\sqrt{u}}$

$\underset{\uparrow \frac{1}{\sqrt{u}} = u^{-\frac{1}{2}}}{=} -\frac{2}{3}u^{\frac{1}{2}}+c = \underline{\underline{-\frac{2\sqrt{1-x^3}}{3}+c}}$

CV : $u = 1-x^3$
$u' = -3x^2 = \frac{du}{dx} \Rightarrow dx = \frac{du}{-3x^2}$

e) $\int tg(x)dx = \int \frac{\sin(x)}{\cos(x)}dx = -\int \frac{\sin(x)}{u}\frac{du}{\sin(x)} = -\int \frac{du}{u} = -\ln(u)+c$

$= \underline{\underline{-\ln(\cos(x))+c}}$

CV : $u = \cos(x)$
$u' = -\sin(x) \Rightarrow dx = \frac{du}{-\sin(x)}$

f) $\int \sin(4x)dx = \int \sin(4x)dx = \underline{\underline{-\frac{1}{4}\cos(4x)+c}}$

Ajustement : u = 4x

g) $\int x\sin(x^2)dx = \frac{1}{2}\int x\sin(u)\frac{du}{x} = \frac{1}{2}\int \sin(u)du = -\frac{1}{2}\cos(u)+c$

$= \underline{\underline{-\frac{1}{2}\cos(x^2)+c}}$

CV : $u = x^2$
$u' = 2x = \frac{du}{dx} \Rightarrow dx = \frac{du}{2x}$

h) $\int \frac{(\ln(x))^2}{x}dx = \int \frac{u^2}{x}x\,du = \int u^2du = \frac{u^3}{3}+c = \underline{\underline{\frac{(\ln(x))^3}{3}+c}}$

CV : $u = \ln(x)$
$u' = \frac{1}{x} = \frac{du}{dx} \Rightarrow dx = x\,du$

i) $\int (1+2x^4)x^3dx = \frac{1}{8}\int u\,du = \frac{u^2}{16}+c = \underline{\underline{\frac{(1+2x^4)^2}{16}+c}}$

CV : $u = 1+2x^4$
$u' = 8x^3 = \frac{du}{dx} \Rightarrow dx = \frac{du}{8x^3}$

j) $\int \frac{dx}{x\cdot\ln(x)} = \int \frac{x\cdot du}{x\cdot u} = \int \frac{du}{u} = \ln(u)+c = \underline{\underline{\ln(\ln(x))+c}}$

CV : $u = \ln(x)$
$u' = \frac{1}{x} = \frac{du}{dx} \Rightarrow dx = x\,du$

4

a $\int 3x(2-x)^{11}dx = \int u'v\,dx = uv - \int uv'dx$

$= -\frac{x(2-x)^{12}}{4} + \frac{1}{4}\int (2-x)^{12}dx = -\frac{x(2-x)^{12}}{4} - \frac{1}{4}\frac{(2-x)^{13}}{13} + c$

$= -\frac{1}{52}(2-x)^{12}(13x+2-x) + c = -\frac{1}{52}(2-x)^{12}(12x+2) + c$

p.p $\left\| \begin{array}{l} u = -\frac{(2-x)^{12}}{12} \underset{\text{ajust}}{\Leftarrow} u' = (2-x)^{11} \\ v = 3x \Rightarrow v' = 3 \end{array} \right.$

b) $\int x^2e^x dx = \int u'v\,dx = uv - \int uv'dx = x^2e^x - 2\underbrace{\int xe^x dx}_{J,\text{cf a)}}$

$= x^2e^x - 2(e^x(x-1)) + c\ \ e^x(x^2-2x+2) + c$

p.p. $\left\| \begin{array}{l} u = e^x \Leftarrow u' = e^x \\ v = x^2 \Rightarrow v' = 2x \end{array} \right.$

c) $\int xe^{x^2}dx = \frac{1}{2}\int xe^u \frac{du}{x} = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2} + c$

CV : $u = x^2$

$u' = 2x = \frac{du}{dx} \Rightarrow dx = \frac{du}{2x}$

d) $I = \int \sin(3x)\cos(x)dx = \int u'v\,dx = uv - \int uv'dx$

$= \sin(3x)\sin(x) - 3\underbrace{\int \sin(x)\cos(3x)dx}_{J}$

p.p. $\left\| \begin{array}{l} u = \sin(x) \Leftarrow u' = \cos(x) \\ v = \sin(3x) \Rightarrow v' = 3\cos(3x) \end{array} \right.$

Calcul de $J = \int \sin(x)\cos(3x)dx = \int u'v\,dx = uv - \int uv'dx$

$= -\cos(3x)\cos(x) - 3\underbrace{\int \sin(3x)\cos(x)dx}_{I}$

p.p. $\left\| \begin{array}{l} u = -\cos(x) \Leftarrow u' = \sin(x) \\ v = \cos(3x) \Rightarrow v' = -3\sin(3x) \end{array} \right.$

$\Rightarrow I = \sin(3x)\sin(x) - 3(-\cos(3x)\cos(x) - 3\cdot I) = \sin(3x)\sin(x) + 3\cos(3x)\cos(x) + 9\cdot I$

$\Rightarrow -8\cdot I = \sin(3x)\sin(x) + 3\cos(3x)\cos(x) \Rightarrow I = -\frac{\sin(3x)\sin(x) + 3\cos(3x)\cos(x)}{8} + c$

5

I

a) $\int \frac{dx}{(x+3)^2} = \int \frac{du}{u^2} = \frac{u^{-1}}{-1} + C = -\frac{1}{u} + C = -\frac{1}{x+3} + C$

C.v. : $u = x + 3$; $u' = 1 = \frac{du}{dx}$

b) $\int e^{1-x}\,dx = \int e^u \frac{du}{-1} = -\int e^u\,du = -e^u + C = -e^{1-x} + C$

C.v. : $u = 1-x$; $u' = -1 = \frac{du}{dx}$

c) $\int x \sin(x)\,dx = \int u'v\,dx = uv - \int uv'\,dx = -x\cos(x) + \int \cos(x)\,dx$
$= -x\cos(x) + \sin(x) + C$

p.p. : $u = -\cos(x)$, $v = x$; $u' = \sin(x)$, $v' = 1$

d) $\int \frac{6}{3x+2}\,dx = \int \frac{6}{u}\frac{du}{3} = 2\int \frac{du}{u} = 2\ln(|u|) + C = 2\ln(|3x+2|) + C$

C.v. : $u = 3x+2$; $u' = 3 = \frac{du}{dx}$

e) $\int 2\,\mathrm{tg}(2x)\,dx = \int 2\,\mathrm{tg}(u)\,\frac{du}{2} = \int \mathrm{tg}(u)\,du$
$= -\ln(|\cos(u)|) + C = -\ln(|\cos(2x)|) + C$

C.v. : $u = 2x$; $u' = 2 = \frac{du}{dx}$

f) $\int \frac{dx}{\sqrt[3]{x}} = \int x^{-\frac{1}{3}}\,dx = \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C = \frac{3}{2}x^{\frac{2}{3}} + C = \frac{3}{2}\sqrt[3]{x^2} + C$

g) $\int (3x-1)e^x\,dx = \int u'v\,dx = uv - \int uv'\,dx$
$= (3x-1)e^x - 3e^x + C = (3x-4)e^x + C$

p.p. : $u = e^x$, $v = 3x-1$; $u' = e^x$, $v' = 3$

h) $\int \frac{x^4+1}{x^2}\,dx = \int \left(x^2 + \frac{1}{x^2}\right)dx = \int (x^2 + x^{-2})\,dx = \frac{x^3}{3} + \frac{x^{-1}}{-1} + C = \frac{x^3}{3} - \frac{1}{x} + C = \frac{x^4-3}{3x} + C$

II

a) $\int 3^{2x}\,dx = \int 9^x\,dx = \int (e^{\ln(9)})^x\,dx = \int e^{x\cdot\ln(9)}\,dx = \int e^u \frac{du}{\ln(9)} = \frac{1}{\ln(9)}\int e^u\,du =$

$\frac{e^u}{\ln(9)} + C = \frac{e^{x\cdot\ln(9)}}{\ln(9)} + C = \frac{9^x}{\ln(9)} + C$

C.v. : $u = x\ln(9)$; $u' = \ln(9) = \frac{du}{dx}$

b) $\int x^5 \ln x\,dx = \int u'v\,dx = uv - \int uv'\,dx = \frac{x^6}{6}\ln(x) - \frac{1}{6}\int x^5\,dx =$

$\frac{x^6}{6}\ln(x) - \frac{x^6}{36} + C = \frac{x^6}{36}(6\ln(x) - 1) + C$

p.p. : $u = \frac{x^6}{6}$, $v = \ln(x)$; $u' = x^5$, $v' = \frac{1}{x}$

c) $\int x^n \ln x\,dx = \frac{x^{n+1}}{n+1}\ln(x) - \frac{x^{n+1}}{(n+1)^2} + C$

p.p. : $u = \frac{x^{n+1}}{n+1}$, $v = \ln(x)$; $u' = x^n$, $v' = \frac{1}{x}$

II -suite

d) $I=\int\frac{8}{x^2-1}dx$

$$=4\int\frac{1}{x-1}dx-4\int\frac{1}{x+1}dx$$

$$=\underline{\underline{4\ln(x-1)-4\ln(x+1)+C}}$$

$$=\underline{\underline{\ln\left(\left(\frac{x-1}{x+1}\right)^4\right)+C}}$$

On veut :

$$\frac{8}{x^2-1}\underset{\text{factorisation}}{=}\frac{8}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}$$

On peut écrire :

$$\frac{A}{x-1}+\frac{B}{x+1}=\frac{A(x+1)+B(x-1)}{(x-1)(x+1)}=\frac{x(A+B)+A-B}{(x-1)(x+1)}$$

Comme : $\frac{8}{(x-1)(x+1)}=\frac{x(A+B)+A-B}{(x-1)(x+1)}$,

on en déduit $\begin{cases}A+B=0\\A-B=8\end{cases}\Rightarrow\begin{cases}A=4\\B=-4\end{cases}$

e) $\int\frac{3x^2-5}{x+3}dx\underset{\text{cf division}}{=}\int\left(3x-9+\frac{22}{x+3}\right)dx$

$$=\int(3x-9)dx+22\int\frac{1}{x+3}dx=\underline{\underline{\frac{3x^2}{2}-9x+22\ln(x+3)+C}}$$

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 3·x²        - 5   :  x + 3
-( 3·x² + 9x     )    3·x - 9
        -9x - 5
      -( -9x - 27 )
             22
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f) $\int\frac{tg(x)}{\cos(x)}dx\underset{tg=\frac{\sin}{\cos}}{=}\int\frac{\sin(x)}{\cos^2(x)}dx\underset{CV}{=}-\int\frac{1}{u^2}du=\frac{1}{u}+C=\underline{\underline{\frac{1}{\cos(x)}+C}}$

C.V:

$u=\cos(x)\Rightarrow u'=-\sin(x)=\frac{du}{dx}$

$\Rightarrow dx=\frac{du}{-\sin(x)}$

g) $I=\int\frac{x^2-x+2}{x^2+2}dx\underset{\text{cf division}}{=}\int\left(1-\frac{x}{x^2+2}\right)dx=\int dx-\underbrace{\int\left(\frac{x}{x^2+2}\right)dx}_{J}$

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 x²   - x + 2   :  x² + 2
-( x²     + 2 )    1
      -x
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Calcul de J : $J=\int\left(\frac{x}{x^2+2}\right)dx=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln(u)=\frac{1}{2}\ln(x^2+2)$

C.V: $u=x^2+2\Rightarrow u'=2x=\frac{du}{dx}$

$\Rightarrow dx=\frac{du}{2x}$

$\Rightarrow I=x-J=\underline{\underline{x-\frac{1}{2}\ln(x^2+2)+C}}$

h) $I=\int\frac{x+2}{x^2-5x+6}dx$

$$=-4\int\frac{1}{x-2}dx+5\int\frac{1}{x-3}dx$$

$$=\underline{\underline{-4\ln(x-2)+5\ln(x-3)+C}}$$

$$=\underline{\underline{\ln\left(\frac{(x-3)^5}{(x-2)^4}\right)+C}}$$

On veut :

$$\frac{x+2}{x^2-5x+6}\underset{\text{factorisation}}{=}\frac{x+2}{(x-2)(x-3)}=\frac{A}{x-2}+\frac{B}{x-3}$$

On peut écrire :

$$\frac{A}{x-2}+\frac{B}{x-3}=\frac{A(x-3)+B(x-2)}{(x-2)(x-3)}=\frac{x(A+B)-3A-2B}{(x-2)(x-3)}$$

Comme : $\frac{x+2}{(x-2)(x-3)}=\frac{x(A+B)-3A-2B}{(x-2)(x-3)}$,

on en déduit $\begin{cases}A+B=1\\-3A-2B=2\end{cases}\Rightarrow\begin{cases}A=-4\\B=5\end{cases}$

III

a) $\int e^x \cos(x)dx = e^x \sin(x) - \int e^x \sin(x)dx = e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x)dx$

car $\int e^x \sin(x)dx = -e^x \cos(x) + \int e^x \cos(x)dx$

Donc $2\int e^x \cos(x)dx = e^x \sin(x) + e^x \cos(x)$ et $\int e^x \cos(x)dx = \dfrac{e^x (\sin(x) + \cos(x))}{2}$

b) $\int \sin(\sqrt{x})\, dx = 2\int u \sin(u)\, du =$ C.v. : $u = \sqrt{x}$: $u' = \dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}} = \dfrac{1}{2u}$

$= \ldots = -2u \cos(u) + 2\sin(u) + c = -2\sqrt{x} \cos(\sqrt{x}) + 2\sin(\sqrt{x}) + c$ cf #c partie I

c) $\int \dfrac{x-1}{x+1}\, dx = \int 1 - \dfrac{2}{x+1}\, dx = \int dx - 2\int \dfrac{1}{x+1}\, dx = x - \ln|x+1| + C$

d) $\int tg^2(x)\, dx = \int \dfrac{\sin^2(x)}{\cos^2(x)}\, dx = \int \dfrac{1-\cos^2(x)}{\cos^2(x)}\, dx = \int \dfrac{1}{\cos^2(x)}\, dx - \int dx = tg(x) - x + C$

e) $\int \ln^2(x)\, dx = x \ln^2(x) - x \ln(x) - \int (\ln(x) - 1)\, dx = x \ln^2(x) - 2x \ln(x) + 2x + C$

f) $\int \dfrac{x^4 + x - 4}{x^2+2} dx = \int (x^2 - 2 + \dfrac{x}{x^2+2}) dx = \int (x^2-2) dx + \int \dfrac{x}{x^2+2} dx = \dfrac{x^3}{3} - 2x + \dfrac{1}{2}\ln(x^2+2) + C$

car $\int \dfrac{x}{x^2+2} dx = \dfrac{1}{2}\int \dfrac{1}{u} du = \dfrac{1}{2}\ln|u| = \dfrac{1}{2}\ln(x^2+2)$

