Lycée Jean-Piaget
Neuchâtel

Mathématiques

Examen de maturité 2013
Corrigé

Exercice 1

Partie 1

$$f(x) = \frac{x^2+2x-3}{x+2} = \dots = \frac{(x+3)(x-1)}{x+2}$$

1. $D = \mathbb{R} \setminus \{-2\}$

2.

		-3		-2		1	
$x+3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x+2$	$-$	$-$	$-$	0	$+$	$+$	$+$
$f(x)$	$-$	0	$+$	∞	$-$	0	$+$

3. A.V. $\underline{x = -2}$

$$\begin{array}{l|l} x^2+2x-3 & x+2 \\ \hline -(x^2+2x) & x + \dfrac{-3}{x+2} \\ \quad\quad\quad\quad -3 & \end{array}$$

A.N.V. $\underline{y = x}$

4. $$y' = \frac{(2x+2)(x+2)-(x^2+2x-3)\cdot 1}{(x+2)^2} = \frac{2x^2+4x+2x+4-x^2-2x+3}{(x+2)^2}$$

$$= \frac{x^2+4x+7}{(x+2)^2}$$

		-2	
x^2+4x+7	$+$	$+$	$+$
$(x+2)^2$	$+$	0	$+$
y'	$+$	∞	$+$
CROISSANCE : y	$\nearrow$	$\parallel$	$\nearrow$

5. $$y'' = \frac{-6}{(x+2)^3}$$

		-2	
-6	$-$	$-$	$-$
$(x+2)^3$	$-$	0	$+$
y''	$+$	∞	$-$
COURBURE : y	$\cup$	$\parallel$	$\cap$

6. $T(1;0)$ $\quad t: y = mx + h$

$m = f'(1) = \frac{12}{9} = \frac{4}{3}$

$y = \frac{4}{3}x + h$ par $T(1;0)$

$\Rightarrow \underline{\underline{t: \; y = \frac{4}{3}x - \frac{4}{3}}}$

7. $G_f \cap Ox = \{(-3;0), (1;0)\}$

$\Rightarrow S = \left| \int_0^1 f(x)\,dx \right| + \int_1^5 f(x)\,dx = S_1 + S_2$

$\int f(x)\,dx = \int \left(x + \frac{-3}{x+2}\right) dx = \frac{x^2}{2} - 3\ln|x+2| + c$

$S_1 = \left| \frac{1}{2} - 3\ln 3 - 0 + 3\ln 2 \right| = -\frac{1}{2} - 3\ln 2 + 3\ln 3 = 0{,}716$

$S_2 = \frac{25}{2} - 3\ln 7 - \frac{1}{2} + 3\ln 3 = 12 - 3\ln 7 + 3\ln 3 = 9{,}458$

$\underline{\underline{S_{tot} \cong 10{,}174}}$

$\left(S_{tot} = \frac{23}{2} + 6\ln 3 - 3\ln 7 - 3\ln 2\right)$

Partie 2

$$f(x) = (2x^2 - 6x) \cdot e^{\frac{1}{2}x} = u \cdot v$$

1. $$f'(x) = u'v + uv' = (4x-6) \cdot e^{\frac{1}{2}x} + (2x^2 - 6x) e^{\frac{1}{2}x} \cdot \frac{1}{2}$$

$$= e^{\frac{1}{2}x} (x^2 + x - 6) = e^{\frac{1}{2}x} (x+3)(x-2)$$

CROISSANCE

		-3		2	
$e^{\frac{1}{2}x}$	+	+	+	+	+
$x+3$	−	0	+	+	+
$x-2$	−	−	−	0	+
f'	+	0	−	0	+
f	↗	—	↘	—	↗

tgtes horizontales en $x = -3$ et $x = 2$

MAX $(-3 ; \ )$ MIN $(2 ; -4e)$

2. $\Rightarrow$ tangente en MIN : $\underline{y = -4e}$

3. a) Pas d'asymptote verticale, $D = \mathbb{R}$

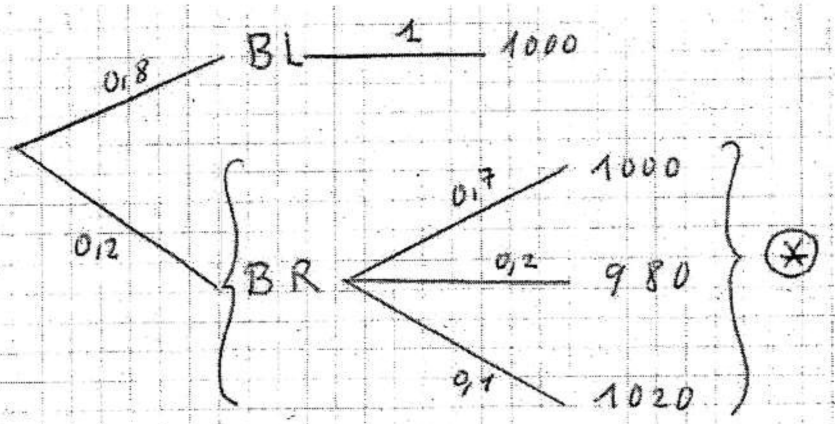
b) $\lim\limits_{x \to \infty} (2x^2 - 6x) \cdot e^{\frac{1}{2}x} = \infty \cdot \infty = \infty$

$\lim\limits_{x \to -\infty} (2x^2 - 6x) \cdot e^{\frac{1}{2}x} = \infty \cdot 0 = 0$

C.E.P.

Asymptote horizontale à g. : $\underline{y = 0}$

Exercice 2



a) $P(\geq 1000) = 0{,}8 \cdot 1 + 0{,}2 \cdot 0{,}7 + 0{,}2 \cdot 0{,}1 = \underline{0{,}96} = 96\,\%$

b) $P(1000 \cap 980) = \big((0{,}8 + 0{,}2 \cdot 0{,}7) \cdot (0{,}2 \cdot 0{,}2)\big) \cdot 2 = \underline{0{,}0752 = 7{,}5}$

c) $P(BL\,BL\,BL\,BL) = (0{,}8)^4 = \underline{0{,}4096 = 40{,}96\,\%}$

d) $P(\ldots) = C_4^6\,(0{,}8)^4 \cdot (0{,}2)^2 = 15 \cdot 0{,}4096 \cdot 0{,}04$

$= \underline{0{,}24576 = 24{,}58\,\%}$

e) $P(E) > 0{,}99 \Rightarrow P(\bar{E}) \leq 0{,}1$

$(0{,}8)^n \leq 0{,}01$

$n \log(0{,}8) \leq -2$

$n \geq \dfrac{-2}{\log(0{,}8)} = \underline{20{,}63}$

Au MIN 21 boîtes

f) $P(BR/1000) = \dfrac{P(BR \cap 1000)}{P(1000)} = \dfrac{0{,}2 \cdot 0{,}7}{0{,}8 + 0{,}2 \cdot 0{,}7} = \underline{0{,}1489 = 14{,}89\,\%}$

g) ⊛ $P(111/Z3) = \dfrac{P(111 \cap Z3)}{P(Z3)} = \dfrac{(0{,}7)^3}{(0{,}7)^3 + (0{,}7 \cdot 0{,}2 \cdot 0{,}1) \cdot 3!}$

$= \underline{0{,}8033 = 80{,}33\,\%}$

h) 10 A 7 B 2 C 1 D

$N = 2 \cdot 10! \cdot 7! \cdot 2! \cdot 1! \cdot 3! = \underline{4{,}389 \cdot 10^{11}}$

Exercice 3

$\vec{v} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\quad K_1(7;6)$ $\quad A(11;9)$ $\quad \overrightarrow{K_1A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$r_1 = \|\overrightarrow{K_1A}\| = 5$

1. $\underline{C_1 : (x-7)^2 + (y-6)^2 = 25}$

2. $\underline{t : 4x + 3y - 71 = 0}$ $\parallel \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ $\quad t^{(n)} : \dfrac{4x+3y-71}{5} = 0$

3. $x^2 + y^2 + 6y + 5 = 0 \Rightarrow C_2 : x^2 + (y+3)^2 = 4$

$\underline{K_2(0;-3) \quad r_2 = 2}$

4. $\underline{\underline{\delta_{MIN}}} = \delta(K_2;t) - r_2 = \left|\dfrac{0-9-71}{5}\right| - 2 = 16 - 2 = \underline{\underline{14}}$

5. $\vec{v} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \Rightarrow u, v : \dfrac{3x+4y+c}{5} = 0$

$\delta(K_1; u,v) = \pm 5 \quad \Rightarrow \quad \dfrac{21+24+c}{5} = \pm 25$

$45 + c = \pm 5 \Rightarrow c = \begin{cases} -70 \\ -20 \end{cases}$

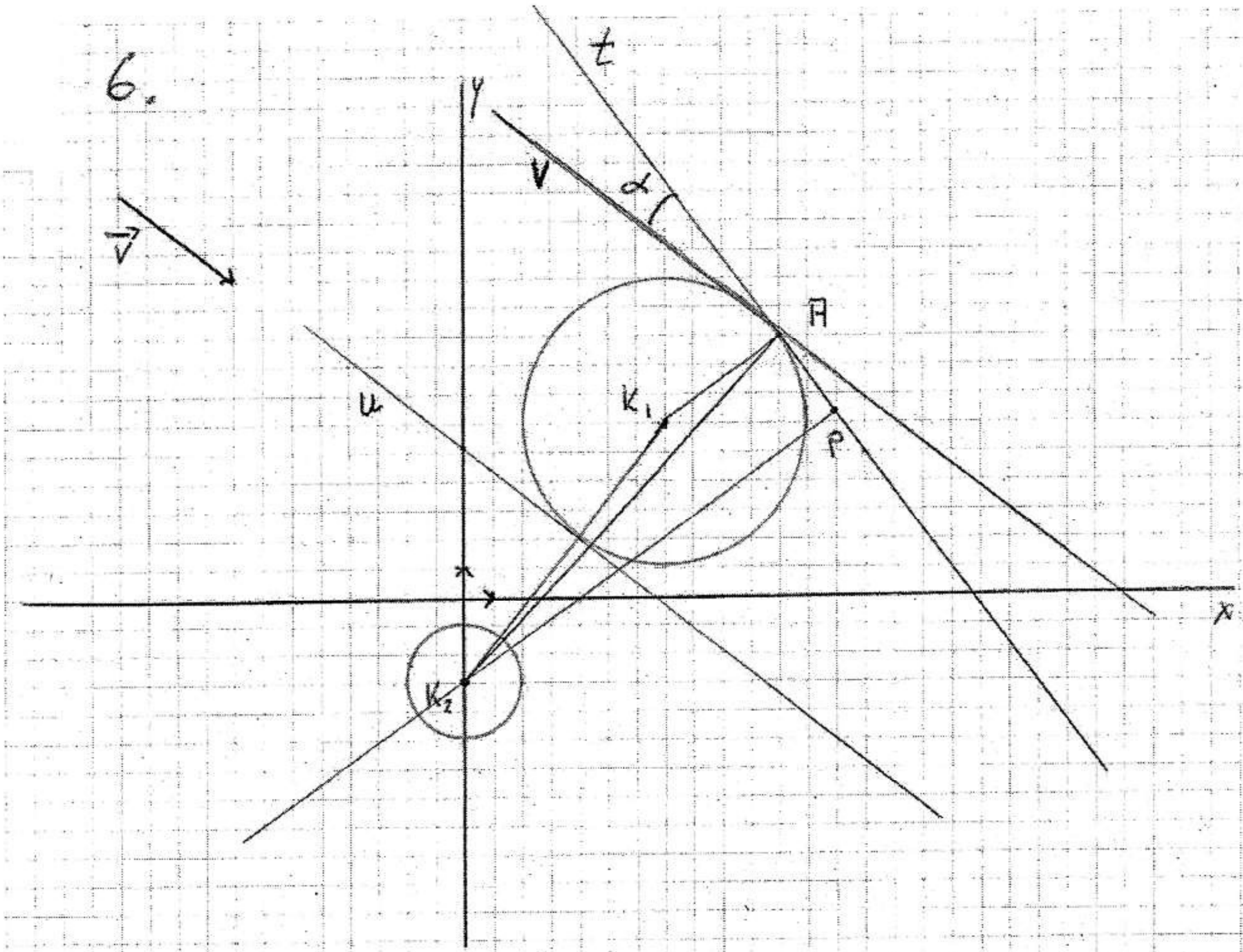
$\Rightarrow \begin{cases} \underline{u : 3x + 4y - 20 = 0} \\ \underline{v : 3x + 4y - 70 = 0} \end{cases} \parallel \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

7. $\cos\alpha = \dfrac{\begin{pmatrix} 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \end{pmatrix}}{25} = \dfrac{24}{25} \Rightarrow \underline{\underline{\alpha = 16{,}26^\circ}}$

8. $S_{AK_1K_2} = \frac{1}{2} D(\overrightarrow{K_1A}; \overrightarrow{K_1K_2}) = \left|\frac{1}{2} \begin{vmatrix} 4 & -7 \\ 3 & -9 \end{vmatrix}\right| = \left|\frac{1}{2}(-36+21)\right| = \underline{\underline{7{,}5}}$

9. $S = c^2 = (\delta(K_2;t))^2 = 16^2 = \underline{\underline{256}}$

6.



10. P est le plus proche de K_2

$$t \perp \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \vec{w} \parallel \vec{i} = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}, \quad \|\vec{i}\| = 1$$

$$\overrightarrow{OP} = \overrightarrow{OK_2} + \overrightarrow{K_2P} = \overrightarrow{OK_2} + 16 \cdot \vec{i}$$

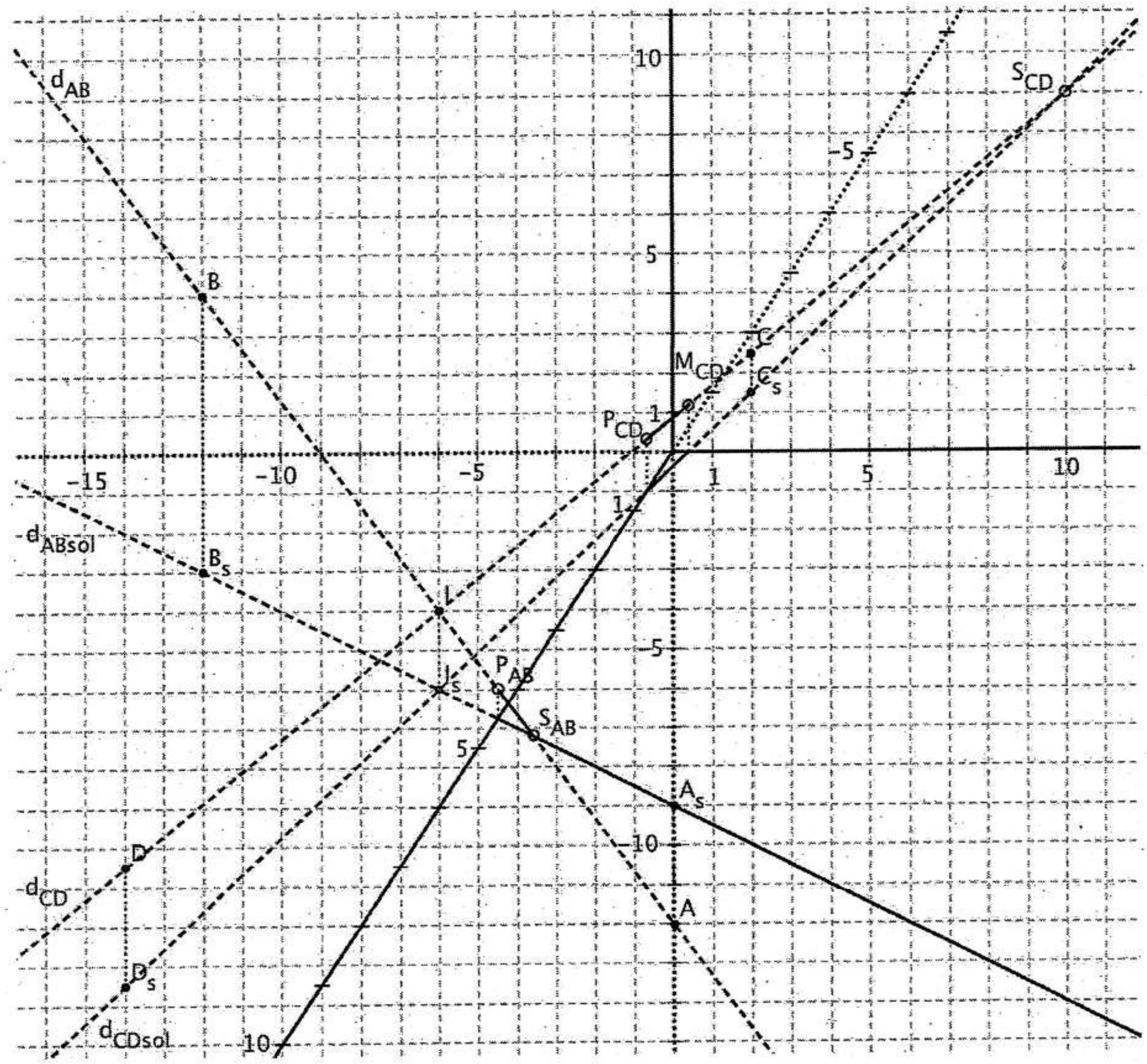
$$= \begin{pmatrix} 0 \\ -3 \end{pmatrix} + \begin{pmatrix} \frac{64}{5} \\ \frac{48}{5} \end{pmatrix} = \begin{pmatrix} 12{,}8 \\ 6{,}6 \end{pmatrix}$$

$$\Longrightarrow \underline{\underline{P\,(12{,}8\,;\,6{,}6)}}$$

ou $P\left(\frac{64}{5}\,;\,\frac{33}{5}\right)$

Exercice 4

Partie 1



Partie 2