1. Pour obtenir la dérivée partielle d'une fonction de plusieurs variables par rapport à une d'entre elles, on fixe les autres variables (on les considère comme constante) et on dérive la fonction par rapport à la variable comme si c'était une fonction d'une variable.

i) $f(x;y) = 4x^3y + x^2y$: $f_x(x;y) = \frac{\partial}{\partial x} f(x;y) = \underline{\underline{12x^2y + 2xy}}$;

$f_y(x;y) = \frac{\partial}{\partial y} f(x;y) = \underline{\underline{4x^3 + x^2}}$.

ii) $f(x;y) = \frac{x^3}{y}$: $f_x(x;y) = \frac{\partial}{\partial x} f(x;y) = \underline{\underline{\frac{3x^2}{y}}}$;

$f_y(x;y) = \frac{\partial}{\partial y} f(x;y) = \underline{\underline{-\frac{x^3}{y^2}}}$.

iii) $f(x;y) = e^{x^2-y^2}$: $f_x(x;y) = \frac{\partial}{\partial x} f(x;y) = e^{x^2-y^2} \cdot 2x = \underline{\underline{2xe^{x^2-y^2}}}$;

$f_y(x;y) = \frac{\partial}{\partial y} f(x;y) = e^{x^2-y^2} \cdot (-2y) = \underline{\underline{-2ye^{x^2-y^2}}}$.

iv) $f(x;y) = \sin(x^2+y^2)$: $f_x(x;y) = \frac{\partial}{\partial x} f(x;y) = \cos(x^2+y^2) \cdot 2x = \underline{\underline{2x\cos(x^2+y^2)}}$;

$f_y(x;y) = \frac{\partial}{\partial y} f(x;y) = \cos(x^2+y^2) \cdot 2y = \underline{\underline{2y\cos(x^2+y^2)}}$.

v) $f(x;y) = \sqrt{x^2+y^2-1}$: $f_x(x;y) = \frac{\partial}{\partial x} f(x;y) = \frac{1}{2\sqrt{x^2+y^2-1}} \cdot 2x = \underline{\underline{\frac{x}{\sqrt{x^2+y^2-1}}}}$;

$f_y(x;y) = \frac{\partial}{\partial y} f(x;y) = \frac{1}{2\sqrt{x^2+y^2-1}} \cdot 2y = \underline{\underline{\frac{y}{\sqrt{x^2+y^2-1}}}}$.

vi) $f(x;y) = \log(\sin(x)\sin(y))$: $f_x(x;y) = \frac{\partial}{\partial x} f(x;y) = \frac{1}{\sin(x)\sin(y)\ln(10)} \cdot \cos(x)\sin(y) =$

$= \frac{\cos(x)}{\sin(x)\ln(10)} = \underline{\underline{\frac{\cot(x)}{\ln(10)}}}$;

$f_y(x;y) = \frac{\partial}{\partial y} f(x;y) = \frac{1}{\sin(x)\sin(y)\ln(10)} \cdot \sin(x)\cos(y) =$

$= \frac{\cos(y)}{\sin(y)\ln(10)} = \underline{\underline{\frac{\cot(y)}{\ln(10)}}}$.

2. i) $f(x;y) = 4x^3y + x^2y$, $f_x(x;y) = 12x^2y + 2xy$, $f_y(x;y) = 4x^3 + x^2$:

$f_{xy}(x;y) = \frac{\partial}{\partial y} f_x(x;y) = \frac{\partial}{\partial y}(12x^3y + 2xy) = \underline{\underline{12x^2 + 2x}}$;

$f_{yx}(x;y) = \frac{\partial}{\partial x} f_y(x;y) = \frac{\partial}{\partial x}(4x^3 + x^2) = \underline{\underline{12x^2 + 2x}}$.

ii) $f(x;y) = \frac{x^3}{y}$, $f_x(x;y) = \frac{3x^2}{y}$, $f_y(x;y) = -\frac{x^3}{y^2}$:

$f_{xy}(x;y) = \frac{\partial}{\partial y} f_x(x;y) = \frac{\partial}{\partial y}\left(\frac{3x^2}{y}\right) = \underline{\underline{-\frac{3x^2}{y^2}}}$;

$f_{yx}(x;y) = \frac{\partial}{\partial x} f_y(x;y) = \frac{\partial}{\partial x}\left(-\frac{x^3}{y^2}\right) = \underline{\underline{-\frac{3x^2}{y^2}}}$.

iii) $f(x;y) = e^{x^2-y^2}$, $f_x(x;y) = 2xe^{x^2-y^2}$, $f_y(x;y) = -2ye^{x^2-y^2}$:

$$f_{xy}(x;y) = \frac{\partial}{\partial y} f_x(x;y) = \frac{\partial}{\partial y}\left(2xe^{x^2-y^2}\right) = 2xe^{x^2-y^2}\cdot(-2y) =$$
$$= \underline{\underline{-4xye^{x^2-y^2}}};$$

$$f_{yx}(x;y) = \frac{\partial}{\partial x} f_y(x;y) = \frac{\partial}{\partial x}\left(-2ye^{x^2-y^2}\right) = -2ye^{x^2-y^2}\cdot 2x =$$
$$= \underline{\underline{-4xye^{x^2-y^2}}}.$$

iv) $f(x;y) = \sin(x^2+y^2)$, $f_x(x;y) = 2x\cos(x^2+y^2)$, $f_y(x;y) = 2y\cos(x^2+y^2)$:

$$f_{xy}(x;y) = \frac{\partial}{\partial y} f_x(x;y) = \frac{\partial}{\partial y}\left(2x\cos(x^2+y^2)\right) = 2x\cdot(-\sin(x^2+y^2))\cdot 2y =$$
$$= \underline{\underline{-4xy\sin(x^2+y^2)}};$$

$$f_{yx}(x;y) = \frac{\partial}{\partial x} f_y(x;y) = \frac{\partial}{\partial x}\left(2y\cos(x^2+y^2)\right) = 2y\cdot(-\sin(x^2+y^2))\cdot 2x =$$
$$= \underline{\underline{-4xy\sin(x^2+y^2)}}.$$

v) $f(x;y) = \sqrt{x^2+y^2-1}$, $f_x(x;y) = \frac{x}{\sqrt{x^2+y^2-1}}$, $f_y(x;y) = \frac{y}{\sqrt{x^2+y^2-1}}$:

$$f_{xy}(x;y) = \frac{\partial}{\partial y} f_x(x;y) = \frac{\partial}{\partial y}\left(\frac{x}{\sqrt{x^2+y^2-1}}\right) = \frac{\partial}{\partial y}\left(x(x^2+y^2-1)^{-\frac{1}{2}}\right) =$$
$$= x\cdot\left(-\tfrac{1}{2}\right)(x^2+y^2-1)^{-\frac{3}{2}}\cdot 2y = \underline{\underline{-xy(x^2+y^2-1)^{-\frac{3}{2}}}};$$

$$f_{yx}(x;y) = \frac{\partial}{\partial x} f_y(x;y) = \frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2-1}}\right) = \frac{\partial}{\partial x}\left(y(x^2+y^2-1)^{-\frac{1}{2}}\right) =$$
$$= y\cdot\left(-\tfrac{1}{2}\right)(x^2+y^2-1)^{-\frac{3}{2}}\cdot 2x = \underline{\underline{-xy(x^2+y^2-1)^{-\frac{3}{2}}}}.$$

vi) $f(x;y) = \log(\sin(x)\cos(y))$, $f_x(x;y) = \frac{\cot(x)}{\ln(10)}$, $f_y(x;y) = \frac{\cot(y)}{\ln(10)}$:

$$f_{xy}(x;y) = \frac{\partial}{\partial y} f_x(x;y) = \frac{\partial}{\partial y}\left(\frac{\cot(x)}{\ln(10)}\right) = \underline{\underline{0}};$$

$$f_{yx}(x;y) = \frac{\partial}{\partial x} f_y(x;y) = \frac{\partial}{\partial x}\left(\frac{\cot(y)}{\ln(10)}\right) = \underline{\underline{0}}.$$

Dans chacun des cas ci-dessus, on a $f_{xy}(x;y) = f_{yx}(x;y)$.

On peut alors décider que $\underline{\underline{f_{xy}(x;y) = f_{yx}(x;y)}}$ de manière générale.