Corrigé

Exercice 1

On peut établir le tableau suivant :

x_i	n_i	$x_i n_i$	x_i^2	$n_i x_i^2$	effectifs cumulés
1	0	0	1	0	0
1,5	0	0	2,25	0	0
2	0	0	4	0	0
2,5	1	2,5	6,25	6,25	1
3	3	9	9	27	4
3,5	5	17,5	12,25	61,25	9
4	4	16	16	64	13
4,5	5	22,5	20,25	101,25	18
5	3	15	25	75	21
5,5	2	11	30,25	60,5	23
6	1	6	36	36	24
Total	24	99,5		431,25	

La moyenne est $\overline{x} = \dfrac{\sum x_i n_i}{\sum n_i} = \dfrac{99,5}{24} \simeq \underline{\underline{4,146}}$.

L'écart type est $\sqrt{\overline{x^2} - \overline{x}^2}$ avec $\overline{x^2} = \dfrac{\sum n_i x_i^2}{\sum n_i} = \dfrac{431,25}{24} \simeq 17,969$ et $\overline{x}^2 \simeq 4,146^2 = 17,188$. Ainsi l'écart-type est $\sqrt{17,969 - 17,188} \simeq \underline{0,884}$.

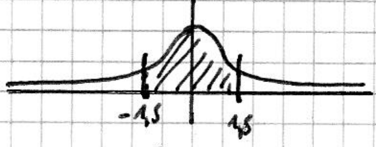
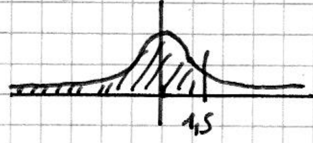
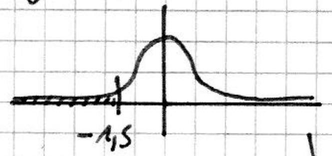
L'effectif total est 24. La médiane correspondra donc à la moyenne entre la 12^e et la 13^e note (à ce point-là, on a exactement 12 notes au-dessous et 12 notes au-dessus de la médiane). La 12^e note est un 4. La 13^e note est aussi un 4. Ainsi, la médiane est $\underline{\underline{4}}$.

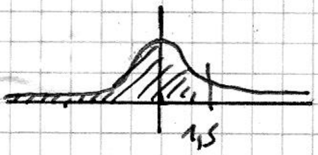
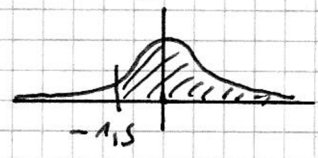
Exercice 2

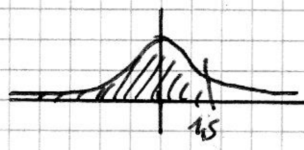
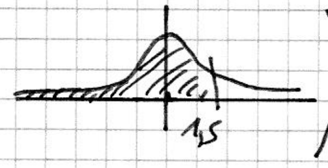
On a $\mu = 223$ gr. et $\sigma = 2$ gr. Notons X le poids d'une baguette.

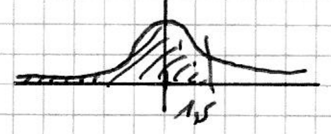
a) $P(220 < X < 226) = P\left(\frac{220-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{226-\mu}{\sigma}\right) =$

$= P\left(\frac{220-223}{2} < \frac{X-\mu}{\sigma} < \frac{226-223}{2}\right) = P\left(-1{,}5 < \frac{X-\mu}{\sigma} < 1{,}5\right) =$

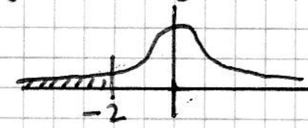
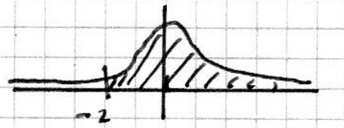
$=$  $=$  $-$  $=$

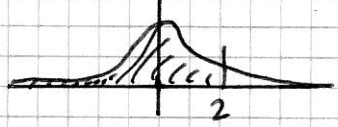
$=$  $-\Big($  $-$  $\Big) =$

$=$  $-\Big(1 -$  $\Big) =$

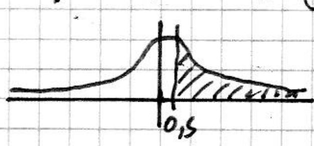
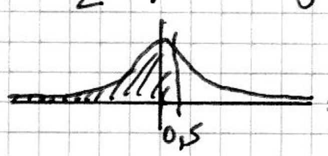
$= 2\cdot$  $-1 = 2\cdot 0{,}93319 - 1 = \underline{\underline{0{,}86638 = 86{,}638\,\%}}$

b) $P(X < 219) = P\left(\frac{X-\mu}{\sigma} < \frac{219-\mu}{\sigma}\right) = P\left(\frac{X-\mu}{\sigma} < \frac{219-223}{2}\right) =$

$= P\left(\frac{X-\mu}{\sigma} < -2\right) =$  $= 1 -$  $=$

$= 1 -$  $= 1 - 0{,}97725 = \underline{\underline{0{,}02275 = 2{,}275\,\%}}$

c) $P(X > 224) = P\left(\frac{X-\mu}{\sigma} > \frac{224-223}{2}\right) = P\left(\frac{X-\mu}{\sigma} > 0{,}5\right) =$

$=$  $= 1 -$  $= 1 - 0{,}69146 = \underline{\underline{0{,}30854}} =$

$= \underline{\underline{30{,}854\,\%}}$

d) C'est une probabilité conditionnelle: $P(A|B) = \frac{P(A\cap B)}{P(B)}$ avec
A = la baguette pèse plus de 225 g et B = la baguette a un poids au-dessus de la moyenne.

On a: $P(A\cap B) = P(X > 225 \text{ et } X > 223) = P(X > 225) =$

$= P\left(\frac{X-\mu}{\sigma} > \frac{225-223}{2}\right) = P\left(\frac{X-\mu}{\sigma} > 1\right) = 1 - P\left(\frac{X-\mu}{\sigma} < 1\right) =$

$= 1 - 0,84134 = 0,15866$ et

$P(B) = P(X > 223) = P\left(\frac{X-\mu}{\sigma} > \frac{223-223}{2}\right) = P\left(\frac{X-\mu}{\sigma} > 0\right) =$

$= 1 - P\left(\frac{X-\mu}{\sigma} < 0\right) = 1 - 0,5 = 0,5.$

Ainsi, la probabilité cherchée est $P(A|B) = \frac{0,15866}{0,5} = \underline{\underline{0,31732 = 31,732\,\%}}$.